

• Why low-power design

① For battery-powered applications where battery life is an important concern.

E.g., PDA, cellular phone, all portable devices.

② High speed circuits.

power dissipation is a problem, e.g., first generation of α chip generates several tens watts of heat.

⇒ increase the cost of packaging (for power dissipation)

⇒ Reduce the life time of the chip.

③ Circuit reliability

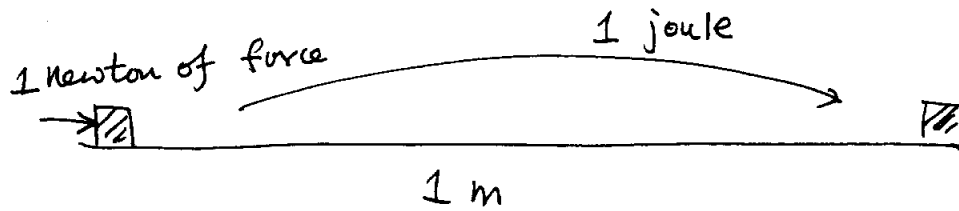
circuit activity is a measure of the "stress" that can cause failures in digital circuits,
run time

power ↑ circuit activity ↑ electromigration, hot carrier ↑ → reliability ↓

∴ should try to reduce power consumption for circuit reliability.

Power Modeling and Sources

- Energy: Represented by joule.



- power: The rate at which work is done.

Represented by Watts.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ sec}}$$

← Consumed or produced

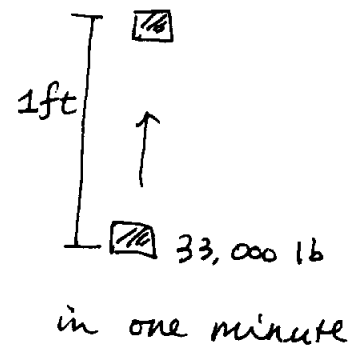
- Relationship between power & energy

$$P (\text{Watts}) = \frac{W (\text{joules})}{t (\text{seconds})}$$

$$P (\text{Watts}) = I (\text{Amp}) \cdot V (\text{voltage})$$

$$E = QV$$

horse power:



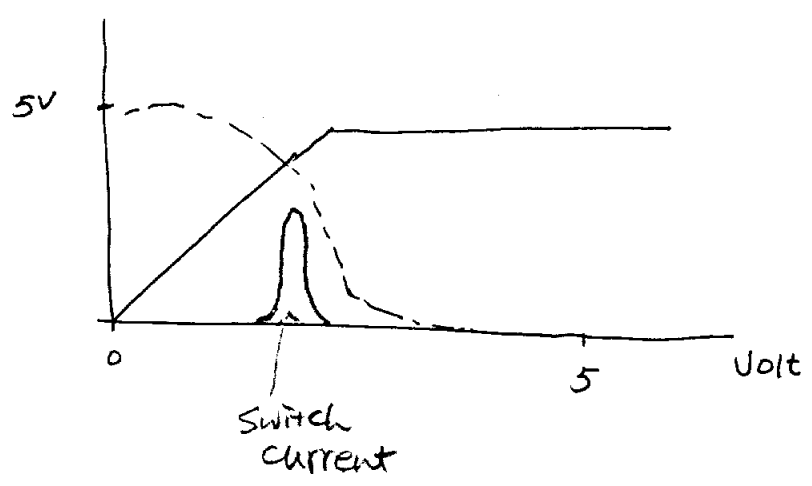
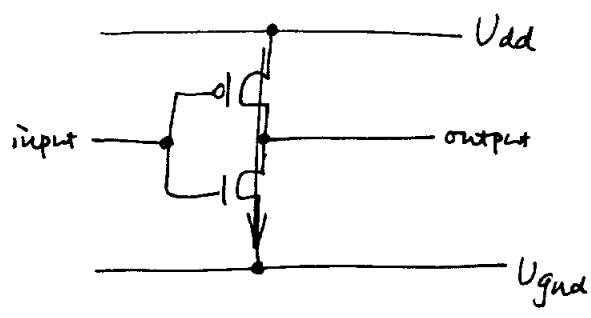
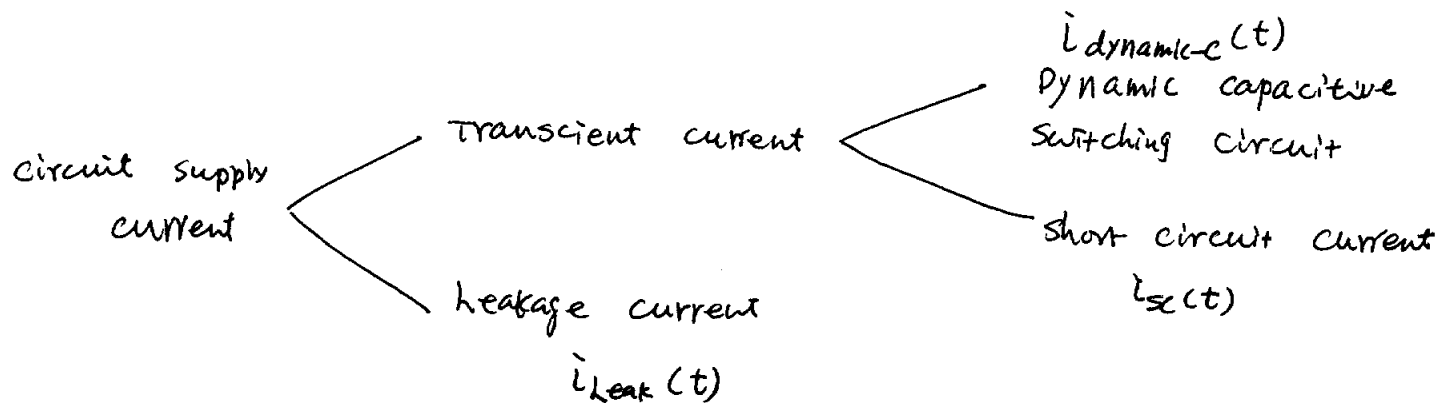
$$1 \text{ hp} = 746 \text{ W.}$$

- Components of power in CMOS.

CMOS: low power device

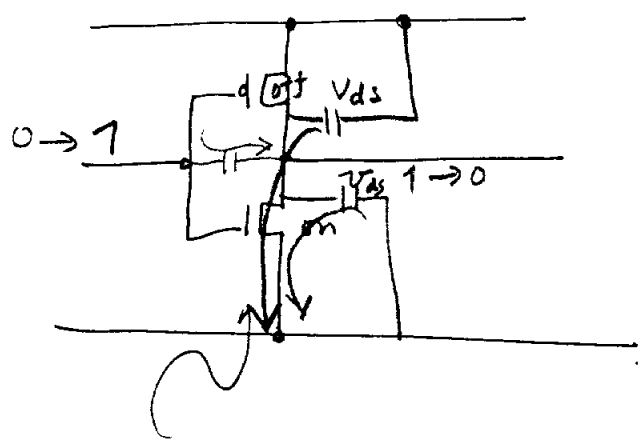
∴ No current flow when inputs & clock are not changing

- * Each signal switch → a certain amount of energy is consumed from the power supply.



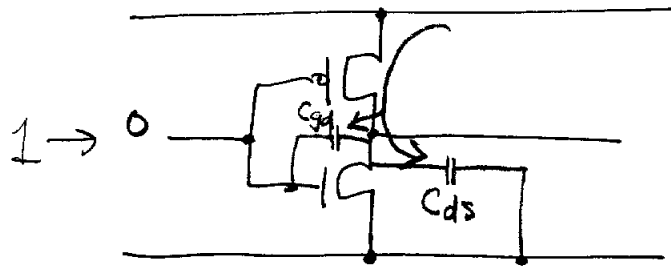
• Current induced by the simultaneous conduction of PMOS & NMOS.

This is called short-circuit current.



* Current required to charge or discharge the capacitors of the ckt.

Dynamic capacitive switching current



$$\dot{i}_{DD}(t) = \dot{i}_{\text{dynamic-c}}(t) + \dot{i}_{sc}(t) + \dot{i}_{\text{Leak}}(t)$$

- The instantaneous power $p(t)$:

$$p(t) = V_{DD} \cdot \dot{i}_{DD}(t)$$

maxi instantaneous power
 \Rightarrow used to estimate the
~~expected life of battery~~
 circuit reliability
 (electromigration, hot carriers)

- The average power P_{avg}

$$P_{\text{avg}} = \left(\frac{1}{T}\right) \int_0^T V_{DD} \cdot \dot{i}_{DD}(t) dt$$

$$= \left(\frac{1}{T}\right) \int_0^T V_{DD} \cdot (\dot{i}_{\text{dynamic-c}}(t) + \dot{i}_{sc}(t) + \dot{i}_{\text{Leak}}(t)) dt$$

\Rightarrow used to estimate the expected life of battery

$$P_{\text{dynamic-c}} = \left(\frac{1}{T}\right) \int_0^T V_{DD} \cdot i_{\text{dynamic-c}}(t) dt$$

Called dynamic power, the major power consumption source.

$$P_{\text{sc}} = \left(\frac{1}{T}\right) \int_0^T V_{DD} \cdot i_{\text{sc}}(t) dt$$

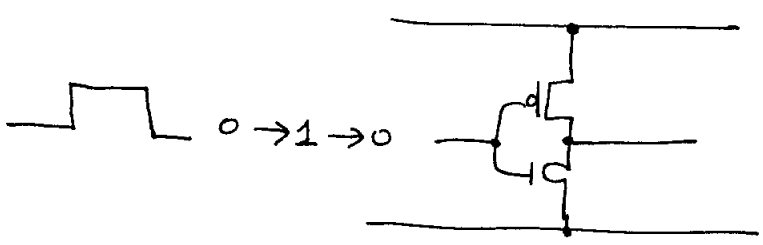
Called short circuit power

$$P_{\text{leak}} = \left(\frac{1}{T}\right) \int_0^T V_{DD} \cdot i_{\text{leak}}(t) dt$$

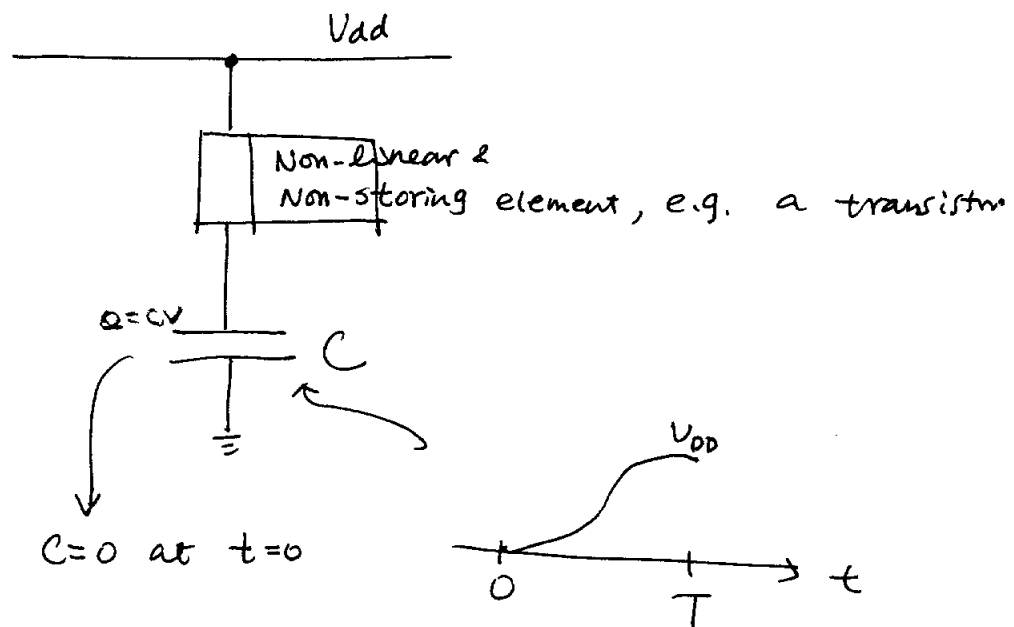
For circuit with long period inactivity \rightarrow this becomes significant

• Dynamic power consumption.

* will analyze



* First, we analyze a simple case



- Energy supplied from V_{DD} .

$$E_{\text{sup}}(T) = V_{DD} \cdot \frac{Q}{C V_{DD}} = C \cdot V_{DD}^2$$

- Energy stored in the capacitor

$$\int_0^{V_{DD}} v(t) d(v) = \frac{1}{2} v(t)^2 \Big|_0^{V_{DD}}$$

$$E_{\text{stored}}(T) = \int_0^Q v(t) dq = \int_0^{V_{DD}} v(t) C dv = \frac{1}{2} C V_{DD}^2$$

$$Q = Cv$$

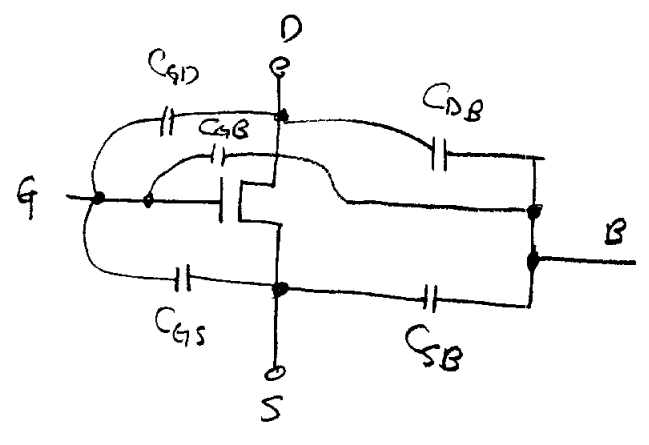
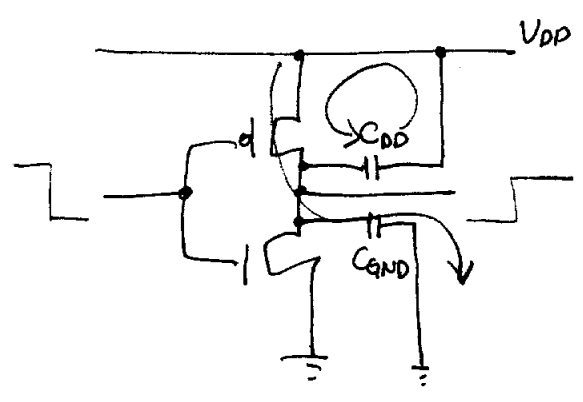
$$dq = C dv$$

- ∴ The rest of energy is dissipated at the non-linear non-storing ~~device~~ as "heat"

$$E_{\text{dis}}(T) = C V_{DD}^2 - \frac{1}{2} C V_{DD}^2 = \frac{1}{2} C V_{DD}^2$$

- ∴ 50% of energy has been lost in "heat"

Load capacitance Model



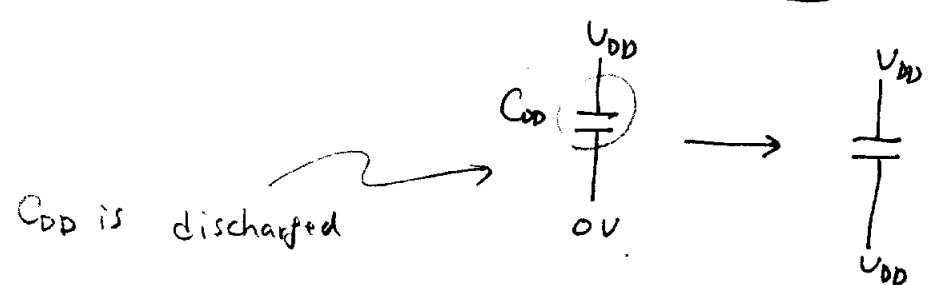
$v_B = 0$ for nMOS
 $v_B = 1$ for pMOS

input: ~~0~~ 1 \rightarrow 0

Energy supplied: $C_{GND} V_{DD}^2$

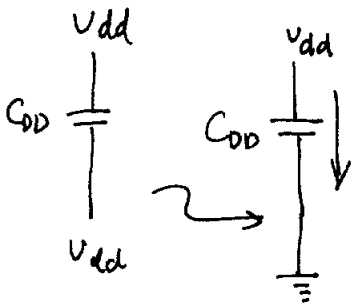
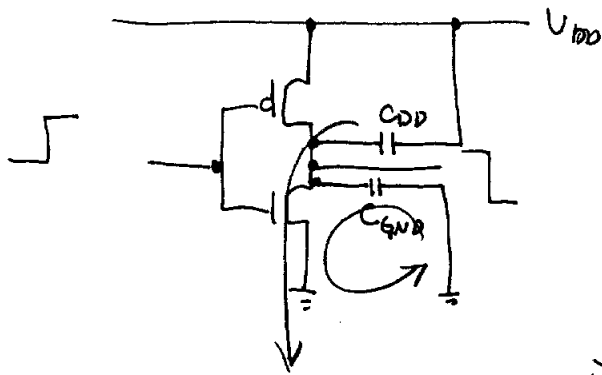
Energy stored in C_{GND} : $\frac{1}{2} C_{GND} V_{DD}^2$

Energy dissipated: $\frac{1}{2} C_{GND} V_{DD}^2 + \frac{1}{2} C_{DD} V_{DD}^2$



C_{DD} is discharged from V_{DD} to 0.

$\therefore \frac{1}{2} C_{DD} V_{DD}^2$ energy saved has been lost!!

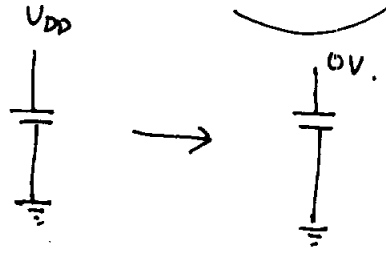


input 0 → 1

Energy supplied: $C_{DD} V_{DD}^2$

Energy stored: $\frac{1}{2} C_{DD} V_{DD}^2$

Energy dissipated: $\frac{1}{2} C_D V_{DD}^2 + \frac{1}{2} C_{GND} V_{DD}^2$



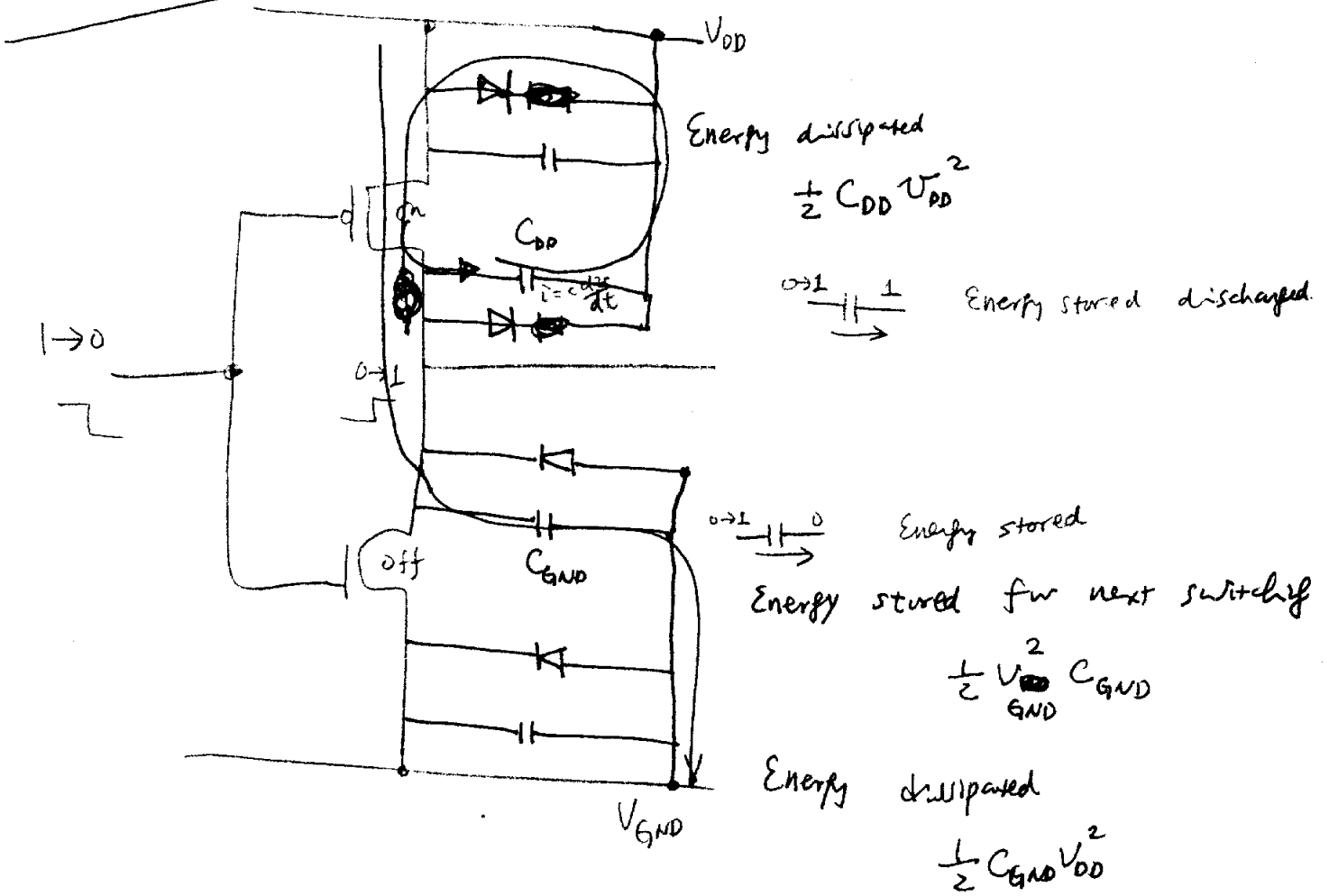
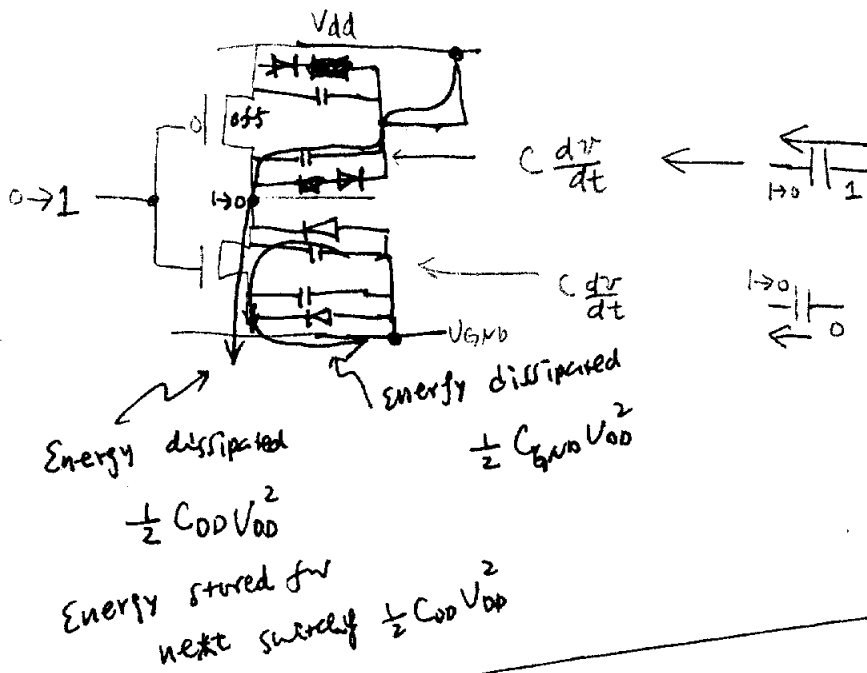
For a Full cycle

0 → 1 → 0

Energy supplied: $(C_{GND} + C_{DD}) V_{DD}^2$

Energy stored: 0

Energy dissipated: $(C_{GND} + C_{DD}) V_{DD}^2$



Input Node transitions

	1 → 0	0 → 1	1 → 0 → 1
Energy supplied	$C_{GND} V_{DD}^2$	$C_{DD} V_{DD}^2$	$(C_{GND} + C_{DD}) V_{DD}^2$
Energy stored	$\frac{1}{2} C_{GND} V_{DD}^2$	$\frac{1}{2} C_{DD} V_{DD}^2$	0
Energy dissipated	$\frac{1}{2} C_{GND} V_{DD}^2 + \frac{1}{2} C_{DD} V_{DD}^2$	$\frac{1}{2} C_{DD} V_{DD}^2 + \frac{1}{2} C_{GND} V_{DD}^2$	$(C_{GND} + C_{DD}) V_{DD}^2$

borrowed from previous state

Consumed by next state

∴ net Energy stored is 0

• Conclusion:

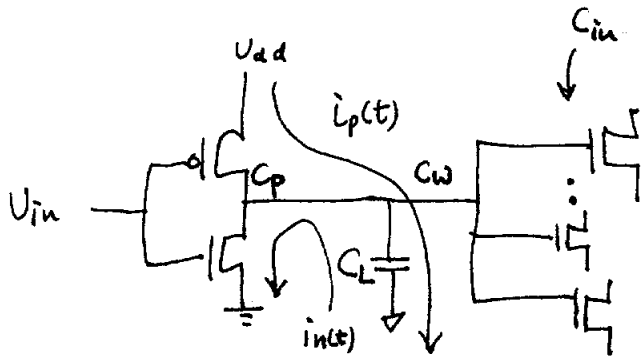
The average dynamic power of a logic node with load capacitance C_L with a complete cycle $0 \rightarrow 1 \rightarrow 0$ (output) transition during time interval T is

$$P_{avg-dynamic} = E_{sup-cycle} / T = C_L V_{DD}^2 / T$$

↑
includes C_{GND}, C_{DD} & C_{load} .

↗
for one period T

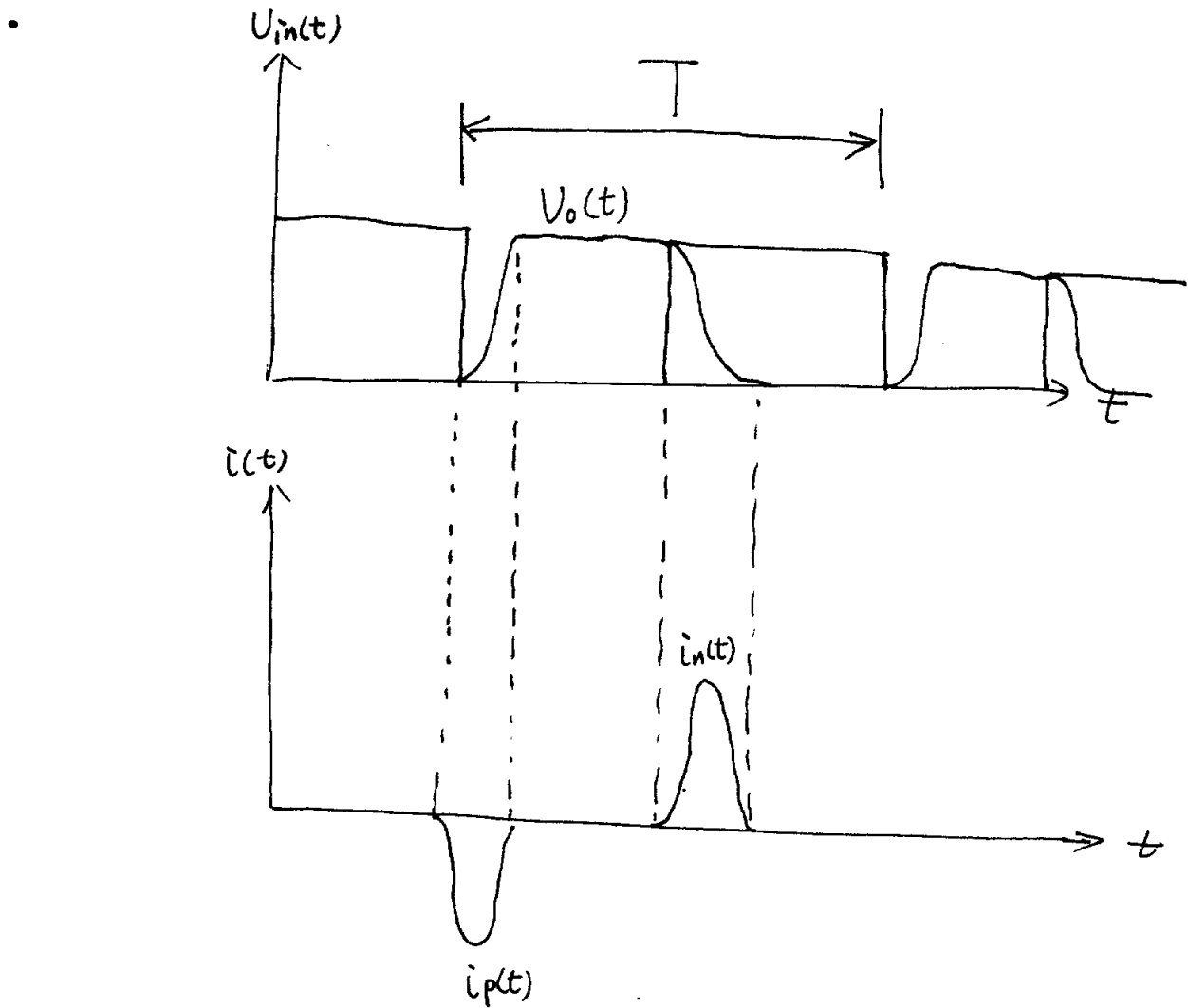
Dynamic Output Load

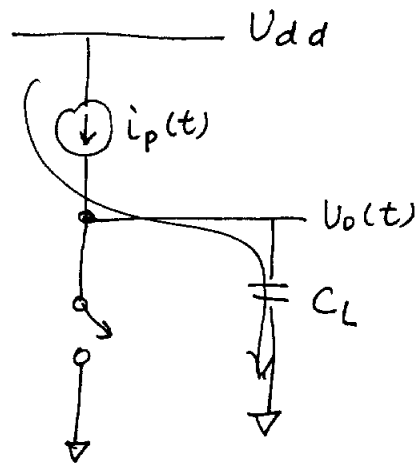
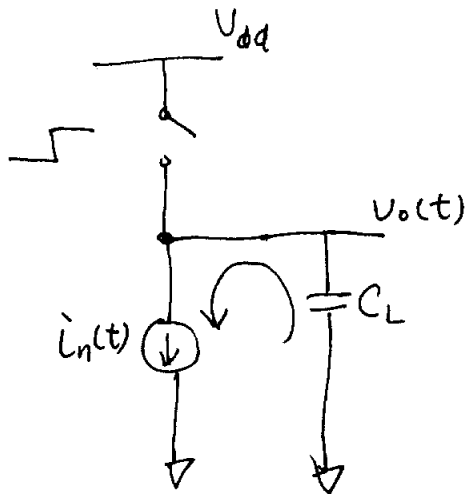


$$C_L = C_{in} + C_w + C_p$$

↑ ↑ ↑
Details will be given later

- C_L is much larger than the parasitic capacitances of the inverters





$$P_d = \frac{1}{T} \int_0^T i_o(t) v_o(t) dt$$

P_d : The average dynamic power required to charge & discharge C_L at frequency $f = \frac{1}{T}$

$$\therefore i_o(t) = i_p(t) = C_L \frac{dv_o}{dt} \leftarrow \text{during charging phase}$$

$$i_o(t) = i_n(t) = -C_L \frac{dv_o}{dt} \leftarrow \text{during discharge phase}$$

$$P_d = \frac{1}{T} \int_0^{V_{dd}} \underbrace{C_L v_o dv_o}_{\text{charge}} - \int_{V_{dd}}^0 \underbrace{C_L v_o dv_o}_{\text{discharge}}$$

$$= \frac{C_L V_{dd}^2}{T} = C_L V_{dd}^2 f$$

Charge phase: output $0 \rightarrow V_{dd}$

Energy drawn from power supply

$$E_d = C_L V_{dd}^2$$

Energy stored:

$$E_{cap} = \int_0^{V_{dd}} C_L V_0 dV_0 = \frac{1}{2} C_L V_{dd}^2$$

Another $\frac{1}{2} C_L V_{dd}^2$ consumed by PMOS transistor

• Discharge phase: output $V_{dd} \rightarrow 0$

No Energy drawn from V_{dd} .

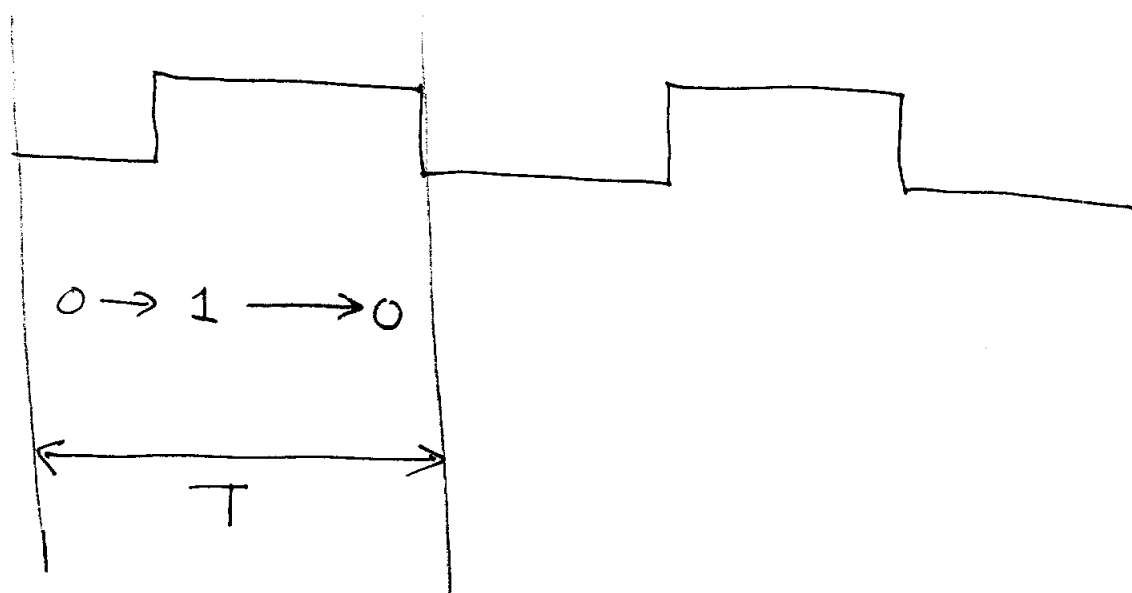
The energy $\frac{1}{2} C_L V_{dd}^2$ stored in C_L is consumed by pull-down NMOS transistor

•• ~~power~~ Energy supplied by V_{dd} : $C_L V_{dd}^2$

~~power~~ Energy consumed
 by PMOS: $\frac{1}{2} C_L V_{dd}^2$
 by NMOS: $\frac{1}{2} C_L V_{dd}^2$

•• ~~power~~ Energy supplied = ~~power~~ Energy consumed.

•• ~~Energy cannot be~~ Energy Conservation



$\alpha_{0 \rightarrow 1}$ = $P(\text{output has a } 0 \rightarrow 1 \text{ transition})$
 node activity

$$P_{\text{ave-dynamic}} = (\alpha_{0 \rightarrow 1} \cdot C_L \cdot V_{DD}^2) / T$$

$$\uparrow$$
 for overall period $= \alpha_{0 \rightarrow 1} \cdot C_L \cdot V_{DD}^2 \cdot f_{CLK}$

definition: $E(SW)$: Expected number of node switching
 $0 \rightarrow 1$ or $1 \rightarrow 0$ during T .

- Node switching
- ~~Number~~ of $0 \rightarrow 1 \rightarrow 0$ is 2
- ~~The # of completed cycles is~~

if $P(0 \rightarrow 1) = P(1 \rightarrow 0)$


$$P(0 \rightarrow 1) = \frac{1}{2} [P(0 \rightarrow 1) + P(1 \rightarrow 0)]$$

$$= \frac{1}{2} [P(0 \rightarrow 1) \cdot 1 + P(1 \rightarrow 0) \cdot 1]$$

$$= \frac{1}{2} E(SW)$$

\therefore ~~the expected # of complete cycles~~ $= \frac{1}{2} E(SW)$
 $\alpha_{0 \rightarrow 1}$

$$\therefore \text{Power-dynamic} = \frac{1}{2} E(\text{sw}) C_L V_{DD}^2 f_{\text{clk}}$$


 $\max [E(\text{sw})] = 2$ if no hazard.

\therefore Key factors influence the dynamic power:

$E(\text{sw})$: linear at each node

C_L : linear

V_{DD} : quadratic \therefore trying to reduce the voltage is the most effective way for low-power design

f_{clk} : linear.

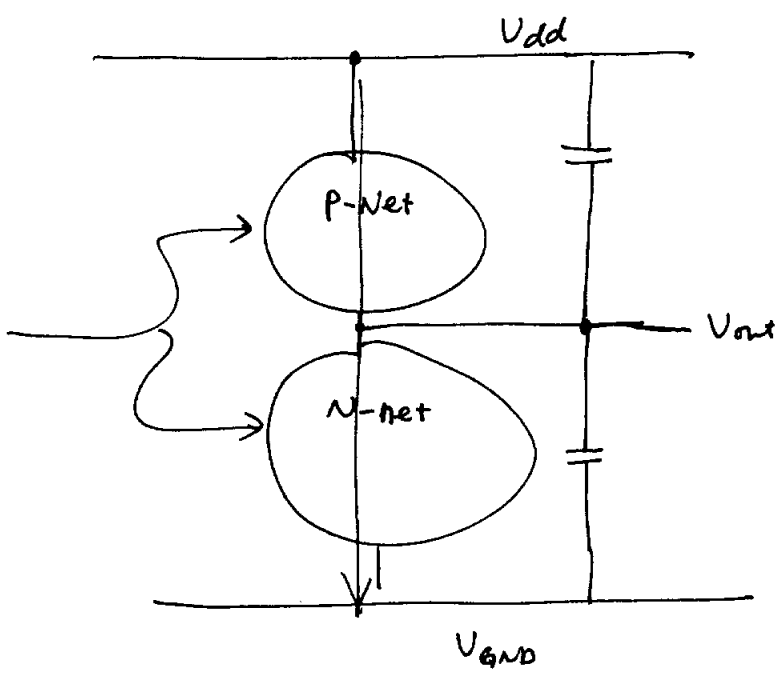
$$\text{Power-dynamic} = \left(\frac{1}{2} E(\text{sw}) C_L \right) V_{DD}^2 f_{\text{clk}}$$

$$= \underline{C_{\text{eff}}} \cdot V_{DD}^2 f_{\text{clk}}$$

Called effective capacitance.

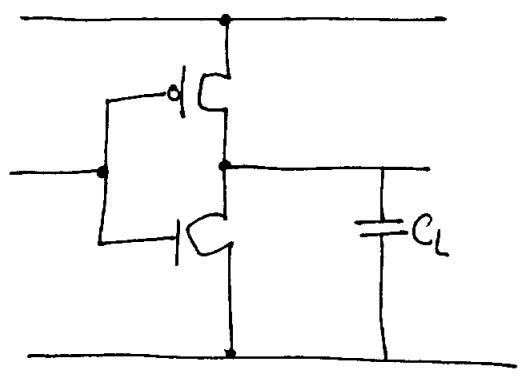
- Short circuit power: Not useful at all.
produces heat in the conducting path.

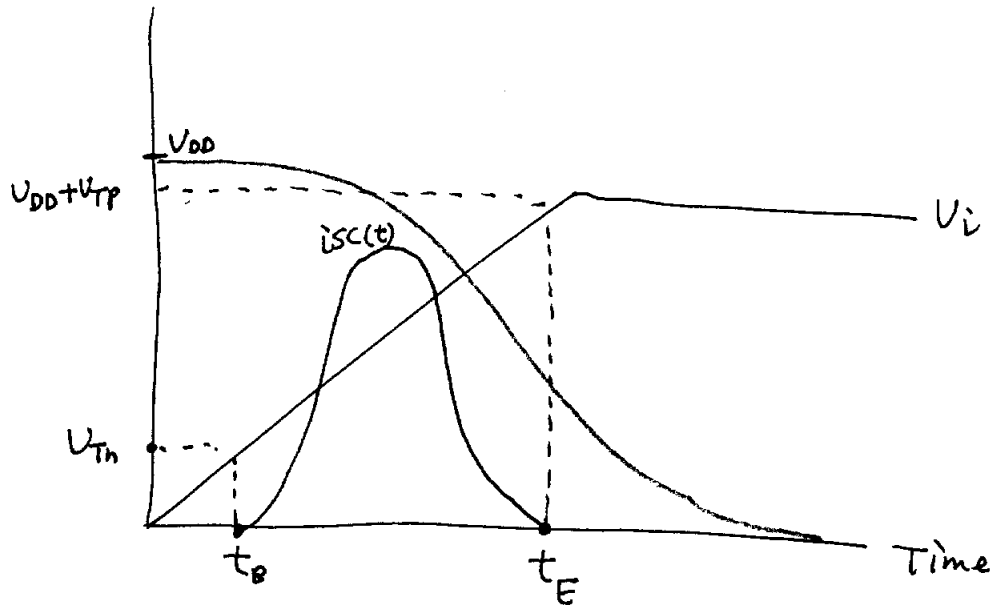
- Short circuit current



The current flowing from V_{dd} to V_{GND} through the the PMOS & NMOS transistor nets for input transitions.

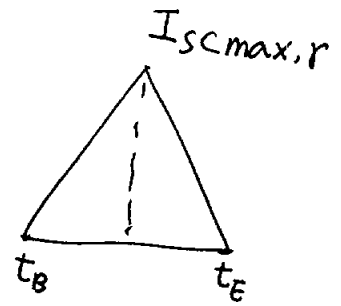
- Show by Inverter





· Rising short circuit Energy

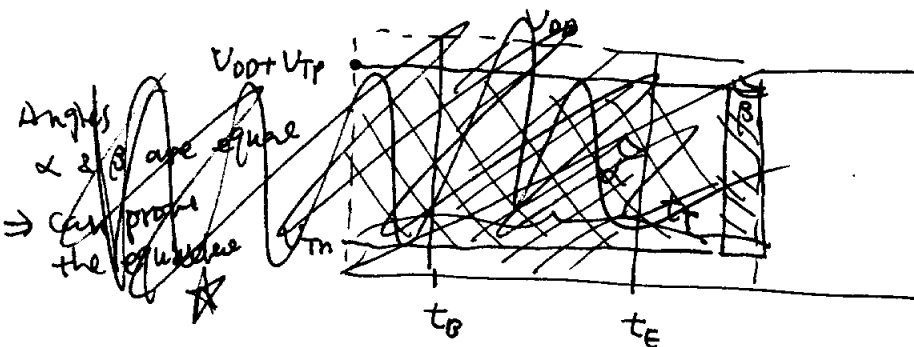
$$E_{scr} = \int_{t_B}^{t_E} V_{DD} \cdot i_{sc}(t) dt \approx$$

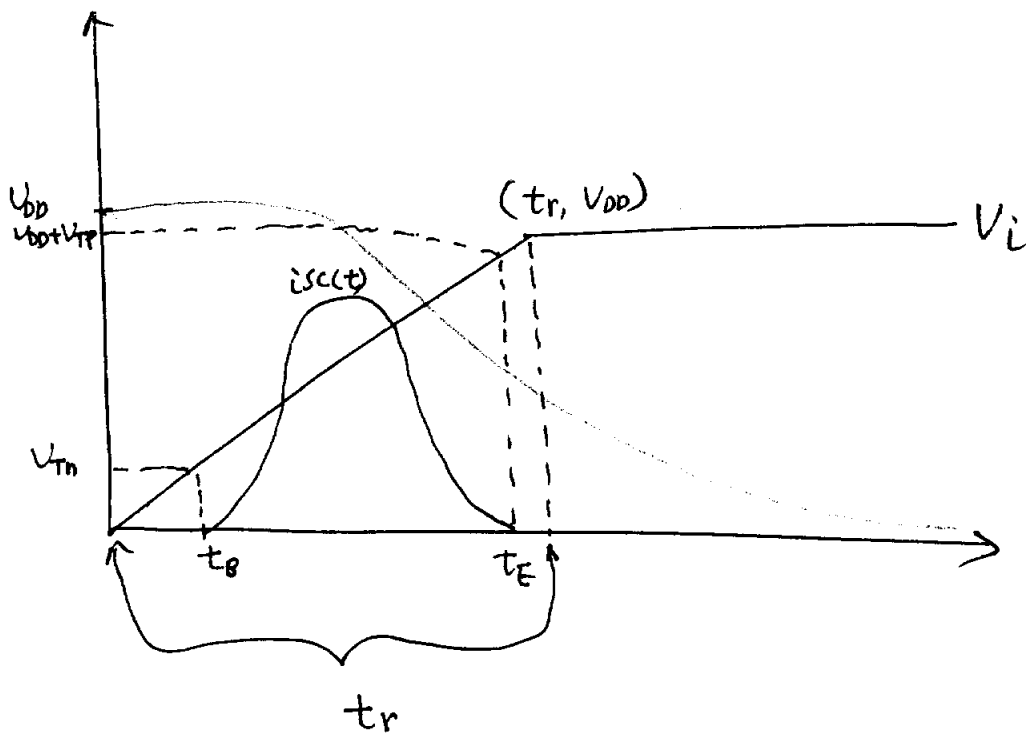


$$\left[\frac{(t_E - t_B)}{2} I_{Scmax,r} \right] \cdot V_{DD}$$

$$= \frac{V_{DD}}{2} (t_E - t_B) I_{Scmax,r} \approx \frac{1}{2} t_r (V_{DD} + V_{TP} - V_{TH}) \cdot I_{Scm}$$

for V_{in}
time required to rise
from 0 to V_{DD}





$$\frac{V_{DD}}{t_r} = \frac{V_{DD} + V_{TP}}{t_E} = \frac{V_{TN}}{t_B}$$

$$\therefore \frac{V_{DD}}{t_r} = \frac{V_{DD} + V_{TP} - V_{TN}}{t_E - t_B}$$

$$\text{Exemple: } \frac{4}{3} = \frac{8}{6} = \frac{20}{15}$$

$$\frac{4}{3} = \frac{12}{9}$$

$$\Rightarrow V_{DD}(t_E - t_B) = (V_{DD} + V_{TP} - V_{TN})t_r$$

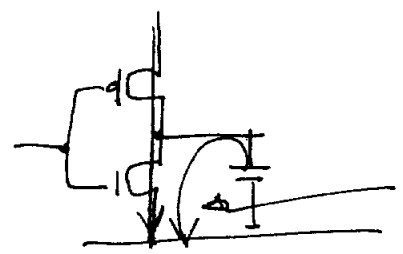
I_{scmaxr} depends on

① The size of the transistors

Larger size \uparrow I_{scmaxr} \uparrow

② Load capacitor C_L :

C_L \uparrow \Rightarrow I_{scmaxr} \downarrow



See next page
larger C_L will induce larger dynamic
capacitive current, reduce the current
flow for I_{scmaxr} .

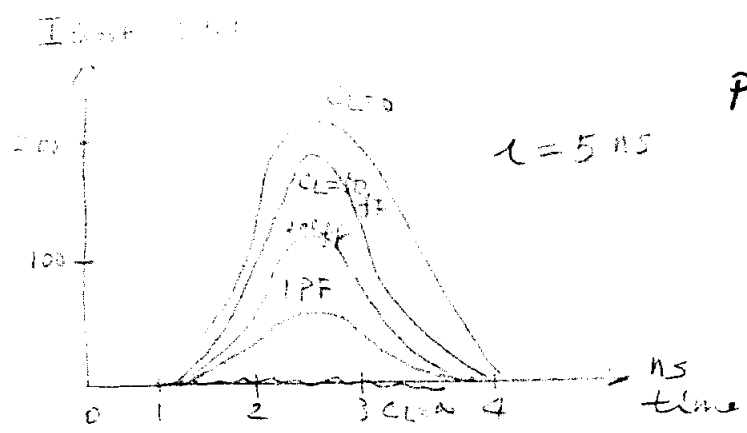
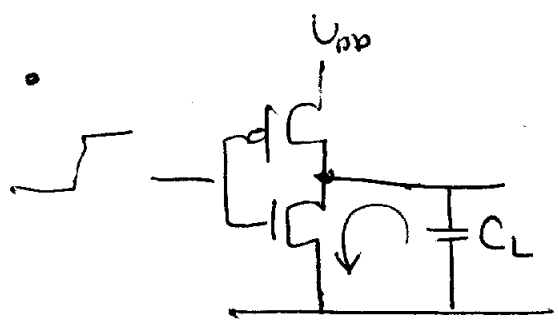
$\therefore I_{scmaxr}$ is max when $C_L = 0$.

• Falling short-circuit energy

$$E_{scf} = \frac{1}{2} t_f (V_{DD} + V_{TP} - V_{TN}) \cdot I_{scmaxf}$$

• Short circuit energy per cycle.

$$E_{sc} = E_{scr} + E_{scf} = (V_{DD} + V_{TP} - V_{TN}) (t_r \cdot I_{scmaxr} + t_f I_{scmaxf}) \frac{1}{2}$$



$C_L \uparrow \Rightarrow I_{SCmax} \downarrow$ Why?

- If C_L is very large, \rightarrow output fall time is much larger than input rise time.
 - V_{ds} of PMOS is essentially 0 during the period
 - Very small I_{sc} .

- If C_L is very small \rightarrow output fall time is much smaller than input rise time.
 - V_{ds} of PMOS is close to V_{dd} during most of the transition time.
 - Very large I_{sc} .

Summary for short ckt energy:

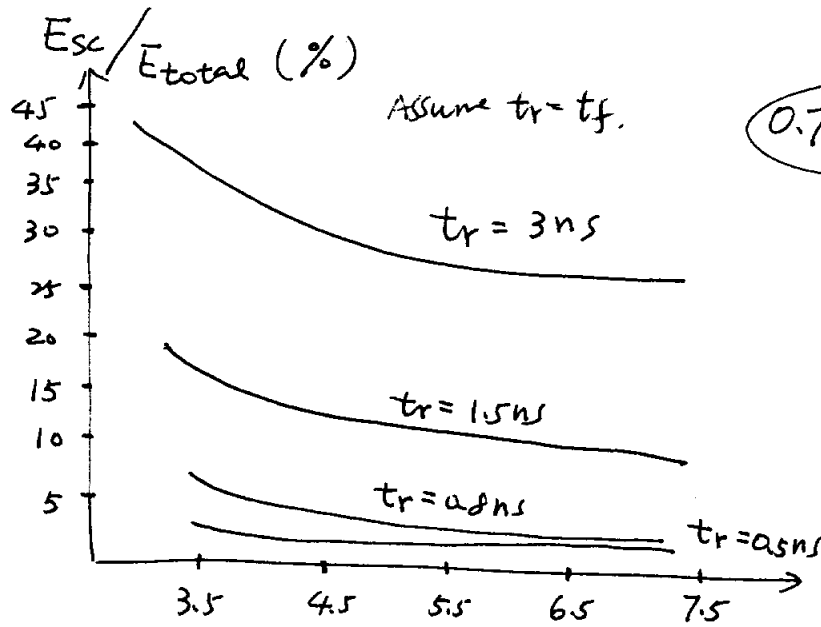
- ① $t_r, t_f \uparrow \Rightarrow E_{sc} \uparrow$
- ② $C_L \uparrow \Rightarrow E_{sc} \downarrow$
- ③ When V_{DD} is scaled down to $\leq |V_{TP}| + V_{TN}$, we have

$E_{sc} = 0.$ (~~no short circuit current~~)

\Downarrow
 $(V_{DD} + V_{TP} - V_{TN} = 0)$
 $\Rightarrow E_{sc} = 0.$

But, the threshold values are not scaled proportionally.

• How important is short circuit energy?



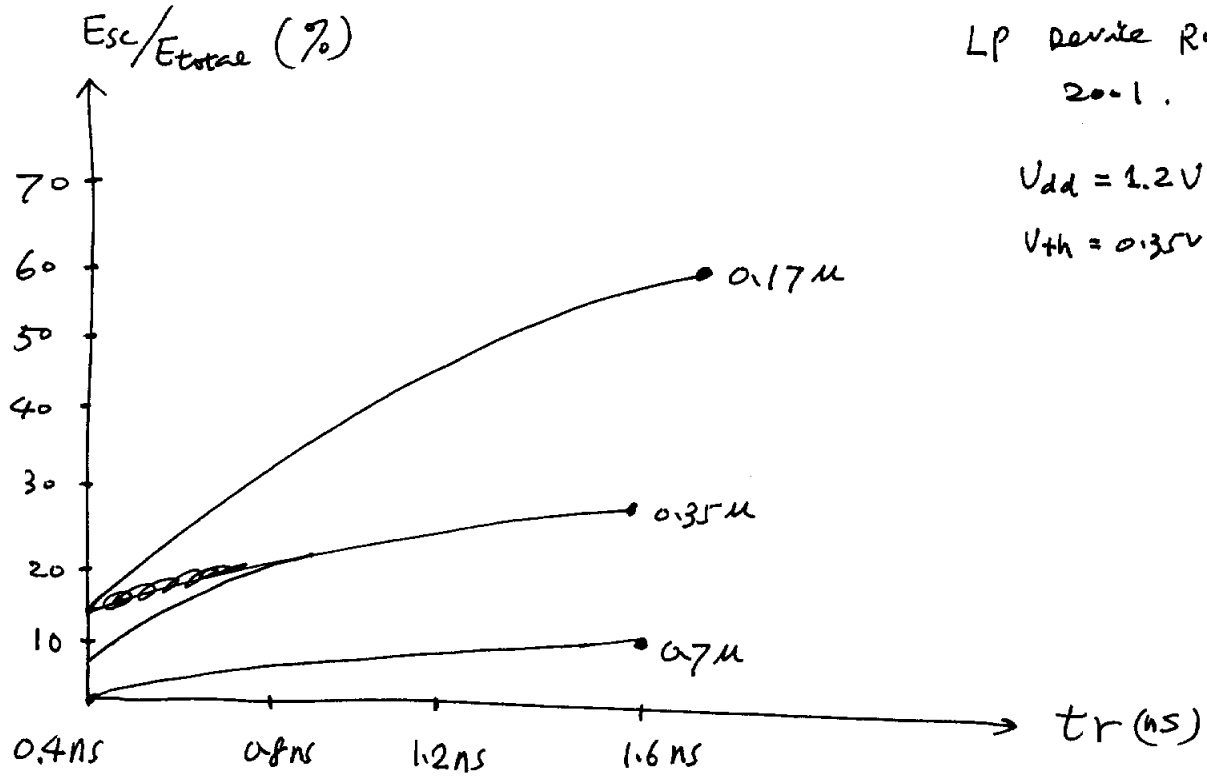
0.7μ technology

below 20% for typical transition time (0.5ns ~ 1.5ns)

$C (F) \times 10^{-14}$
Capacitance loading

$E_{sc} \propto f(t_r, C_L)$

• Effect of scaling



LP device Roadmap 20-1.

$V_{dd} = 1.2V$

$V_{th} = 0.35V$.

$E_{sc} \propto f(t_r, \text{technology})$

0.7μ → $E_{sc}/E_{total} : 1\% \sim 16\%$

0.35μ → $4\% \sim 37\%$

0.17μ → $12\% \sim 60\%$

Main reason:

The threshold Voltage does not scale down proportionally to supply voltage!!!

• Short circuit power model

Average short circuit power

$P_{avgsc} = (a_{0 \rightarrow 1} E_{sc}) / T = a_{0 \rightarrow 1} E_{sc} f_{clk}$

$a_{0 \rightarrow 1}$: Node activity

$P(0 \rightarrow 1 \text{ transition in a single period } T)$.

$$P_{avgSC} = \frac{1}{2} E(SW) \cdot E_{sc} \cdot f_{clk} \quad \text{where } E(SW): \text{ average \# of transitions } 0 \rightarrow 1 \text{ or } 1 \rightarrow 0$$

$$= \frac{1}{2} E(SW) \left[(V_{DD} + V_{TP} - V_{TN}) (t_r \cdot I_{scmaxr} + t_f \cdot I_{scmaxf}) \right] \cdot f_{clk}$$

$$= \frac{1}{4} E(SW) \left[(V_{DD} + V_{TP} - V_{TN}) (t_r \cdot I_{scmaxr} + t_f \cdot I_{scmaxf}) \right] \cdot f_{clk}$$

Summary: P_{avgSC}

- ① proportional to f_{clk} , the clock frequency
- ② linear to the sum of the average transition energy of each node $\frac{E_{sc}}{2}$
- ③ linear to the expected # of switchings.

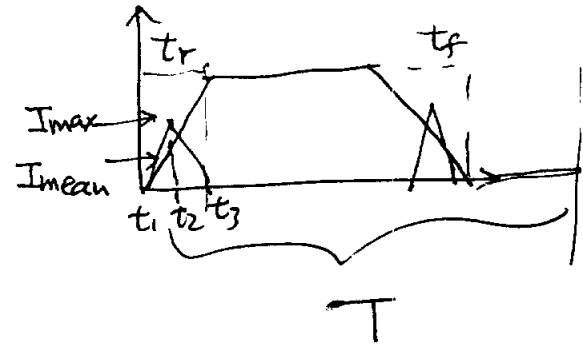
• Analysis can be done by further extending $I_{sc \max}$. P17'

Assume $C_L = 0$ in which I_{sc} is maximum

$$I_{sc}^{(\max)} = \frac{\beta_N}{2} (V_{GS} - V_{TN})^2$$

when saturate

$$\beta_N = \frac{\mu_n C_{ox}}{2} \frac{W}{L}$$



Assume $\beta_N = \beta_P$ and $V_{TN} = |V_{TP}|$

we have

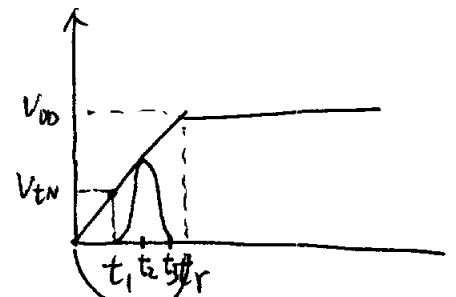
$$I_{sc}(t) = \frac{\beta_N}{2} (V_{in}(t) - V_{TN})^2$$

$$I_{mean} = \frac{2}{T} \cdot 2 \int_{t_1}^{t_2} I_{sc}(t) dt$$

$$= \frac{4}{T} \int_{t_1}^{t_2} \frac{\beta_N}{2} (V_i(t) - V_{TN})^2 dt$$

$$= \frac{\beta_N T}{12 V_{DD} T} (V_{DD} - V_{TN})^3$$

$$\therefore P_{sc} = I_{mean} \cdot V_{DD} = \frac{\beta_N T f}{12} (V_{DD} - V_{TN})^3$$



$$\frac{V_{TN}}{t_1} = \frac{V_{DD}}{t_r}$$

$$\therefore t_1 = \frac{V_{TN} \cdot t_r}{V_{DD}}$$

$$\frac{V_i(t)}{t} = \frac{V_{DD}}{t_r} \therefore V_i(t) = \frac{V_{DD}}{t_r} \cdot t$$

input rise time
input fall time

$$t = t_r = t_f$$

$$V_i(t) = \frac{V_{DD}}{t_r} \cdot t$$

$$t_1 = \frac{V_{TN}}{V_{DD}} \cdot t \quad t_2 = \frac{t}{2}$$

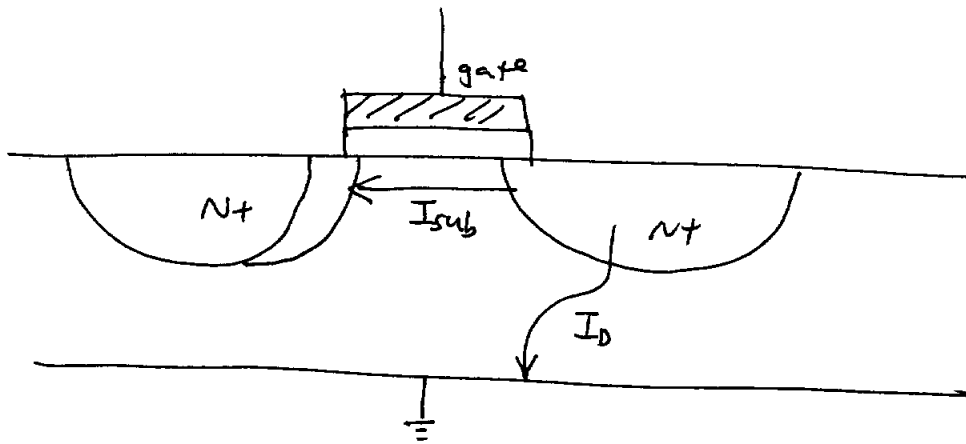
- $P_{sc} \propto f$
- $P_{sc} \propto V_{DD}^3$
- P_{sc} : Designer can just control β and τ .

◦◦ Try to keep rising & falling edges fast!!!

τ)
↓

Leakage power

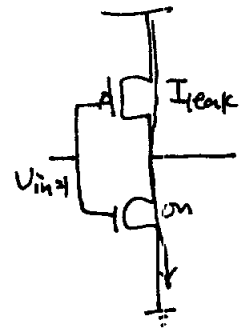
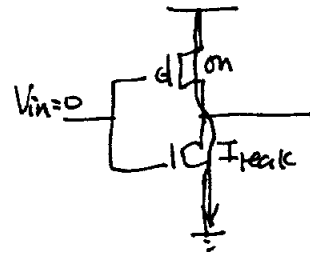
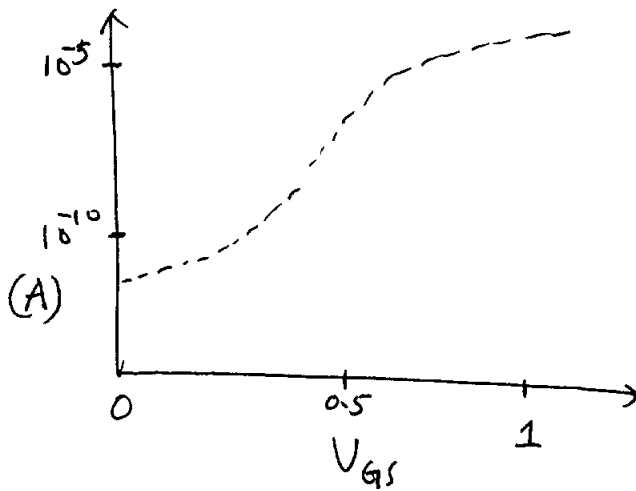
- * A small quiescent leakage current flows in CMOS from V_{dd} to V_{gnd} in static condition (inputs & internal nodes have constant values).
- * $I_{leak} \rightarrow$ leakage power consumption (static)
- * P_{leak} was negligible in old technologies
- * With deep submicron scaling, I_{leak} & P_{leak} are more and more significant.



will discuss two major leakage sources:

- ① subthreshold leakage current
- ② Reverse biased leakage current.

- Subthreshold leakage current — unexpected current flows from drain to source.



$$I_{sub} = \mu_0 C_{ox} \frac{W}{L} \cdot v_t^2 e^{\frac{V_{GS} - U_{TH}}{n \cdot v_t}}$$

μ_0 : carrier surface mobility

C_{ox} : gate oxide capacitance / unit area

W .

L .

v_t : thermal voltage. \propto (T : (absolute temperature)
 $v_t = \frac{kT}{q} = 26 \text{ mV at } 300 \text{ K}$ K : Boltzmann constant.
 ~~n : a technology~~
 q : charge of electron)

n : A technology-dependent parameter

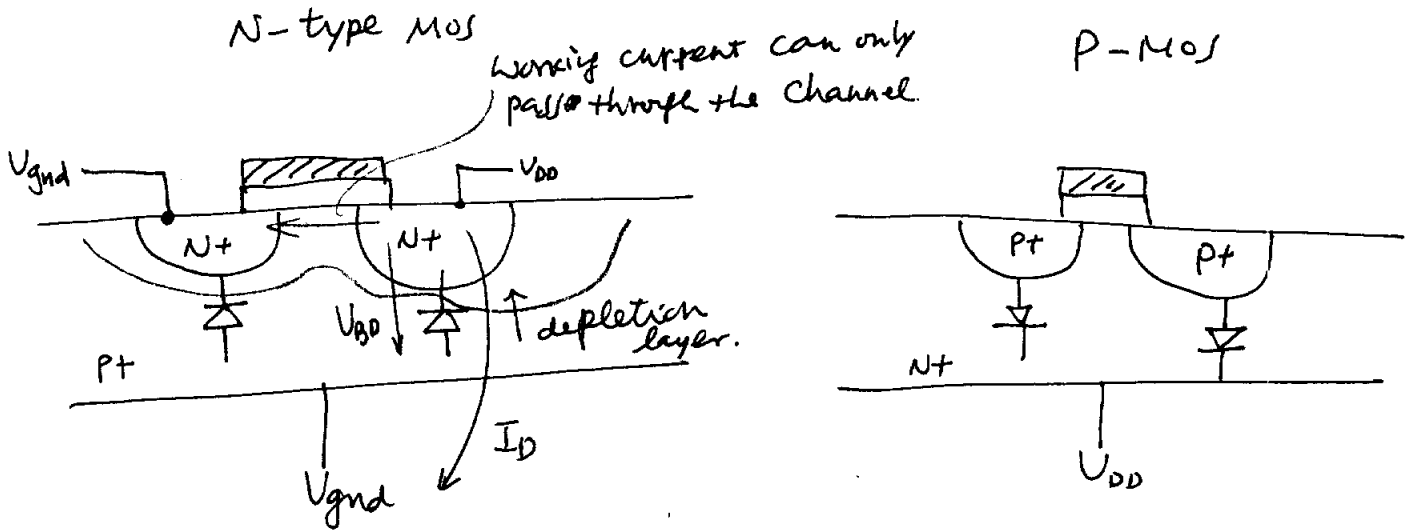
V_{GS} :

U_{TH} : threshold voltage.

Technology advanced $\Rightarrow V_{DD} \uparrow \Rightarrow V_{TH} \uparrow$ (for performance)
 $\Rightarrow I_{sub} \uparrow$

\therefore for deep submicron VLSI, I_{sub} contribution to the leakage power grows in importance.

• Reversed-Biased leakage current



• Diodes are reversely biased

* I_D is related to temperature

$$T \uparrow \rightarrow I_D \uparrow$$

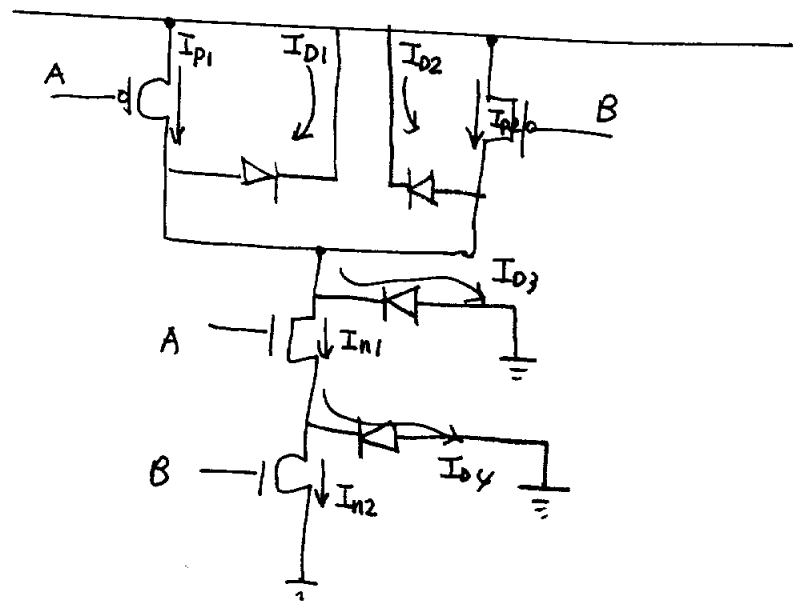
$$I_{D0} = I_s \cdot \left[e^{\frac{q|V_{D0}|}{KT}} - 1 \right]$$

I_s : Saturation current of ~~the~~ p-n junction.

q : charge of electron
 K : Boltzmann constant
 T : Temperature (in terms of absolute temp.)

* In the level of fA (10^{-15} ampere) for 0.35 μ technology.

Example:



• The state and input of a ckt have influence on the leakage power

$I_{sub} \propto V_{GS}$

$I_{BD} \propto V_{BD}$

$I_{BS} \propto V_{BS}$

∴ different circuit state gives different leakage current

Example:

