Nonlinear and Time-varying Systems Analysis, Design, & applications: A Building Block Approach

Motivation:

- Presently NLTV systems are approached using a highly analytical approach;
- Most of the results are based on abstract models such as canonical models;
- Use abstract nonlinear mappings driven from analytical considerations (polynomial non-linearity as in Volterra-Wiener expansion);
- Or very special circuits (such as Chua's chaotic circuit for chaos);
- Overly specialized as separate islands where focus is on a single application (Ex: Nonlinear Control, Chaos etc.), understandable given the complexity
- Appeals to those who are mathematically gifted - mostly at the graduate / academic level
- Leaves out many undergraduate and practicing engineers who should be putting that knowledge to everyday use, as most real-world systems are NL and or TV.

The New Approach:

- A globalistic approach that can be applied to many areas of NLTV systems
- Approached from a practical perspective;
- Starts with the definition of electrical elements needed for building such systems;
- Obviously they are going to be complex as we deal with a complex area but connection to the physical world makes it easy to understand;
- How to interconnect them;
- Extracting the general form of I/O models resulting from such circuits with the elements' characteristics as the primitives;
- Consideration of specific (but general) forms of circuits and their behavior;
- The use of such models in designing systems in a number of applications in the analog as well as the digital domain.
- The analog world helps to overcome big problems such as stability; Digital implementation as the world is becoming digital!

Precedence:

- Synthesis by analysis commonly and successfully used in 1950's Network and System Theory;
- PR (positive real) functions etc. arrived from circuit considerations;
- Such concepts (like positive real) have been used by others, but sort of at a block box or macro level, whereas here they are used at the micro or the element level;
- Lyapunov used a nonlinear mechanical system in developing his theory.

Differences / Advantages:

- A bottom up approach (elements to dynamics to explanation and applications) as opposed to a top-down approach (model to analysis);

- Rather than using non-linear primitives chosen from an analytical perspective, here we choose them based on the elements being used and the general restrictions on them.
- Makes the classification of NL and TV circuits, dynamics more easier;
- Analog representation to handle complex issues such as stability etc;
- Digital representation for easy of implementation.

Elements:

LTI elements summary

NLTI elements:

- **One-port Elements:**

  Nonlinear Resistor:
  - Using one voltage-current waveform, emphasize concepts such as:
  - No-memory;
  - Non-monotonous characteristics (many to one mapping or non-invertible – either voltage controlled or current controlled);
  - Can exist in 2nd and or 4th quadrants (globally passive Vs locally passive);
  - Need not be bilateral;
  - From a practical perspective, to pass through the origin,
  - No discontinuity or one-to-many mappings;
  - Smooth (differentiable) mappings unless demanded by specific applications (chaotic systems, for example);
  - Caution in the use of no linearity which allows almost infinite mappings and hence care should be exercised in their selection (for Ex: $v_r(t) = i_r(t) / \left(1 + i_r^2(t)\right)$ with a diminishing power absorption capacity as the input magnitude becomes large);
  - Use of such elements can lead to dynamics whose equilibrium can be asymptotically stable but may not be under external disturbance.

  Nonlinear Capacitor:
  - Discussion similar to NLR.
  - Element with memory; non-bilateral; charge or voltage controlled;
  - When combined with restrictions on interconnection and solvability of equations, charge controlled capacitors lead to more complex responses (something different from what we learnt in fundamental physics etc. where we consider voltage to be the source or input and the charge as the response or the output);
  - Energy storage and return capacity (which is more complicated than LTI capacitors since the energy waveform can be more complex); Unlike LTI capacitor, may not give back all stored energy (Ex: $v_c(t) = q(t) / \left(1 + q^2(t)\right)$) or state variable may escape to infinity;

- Concept of relaxation points; local minima and maxima on the energy waveform;
- Connection between relaxation and energy-mimina points and stable / unstable / multiple equilibrium points;
- Why stability of equilibrium points and no longer stability of the system or the dynamics

Nonlinear Inductor:

- Use duality principle with the equations/description for NLTI capacitor;

- **Multi-port NLTI Elements:**

  2-port Transformer with a changing turns ratio:

  - The turns ratio can be a function of the state variables alone leading to NLTI element;
  - Or explicit function of time as well (NLTV);
  - Lossless, memoryless (or with memory if we use past history of the state to change the turns ratio) element;
  - Easy to implement digitally;

  m-port (m>=2) gyrators: Similar to transformer discussion; once again easy to implement digitally;

  m-port (m>=2) NLTI mutual Inductors:

  - Energy storage element;
  - Once again, flux controlled leads to more general element as opposed to current controlled;
  - Coupling between state variables lead to more complex energy storage functions;

  m-port NLTI mutual capacitors:

  - Use duality to define;
  - Even though physically may or may not be possible in the analog world, no restriction in the digital world;

NLTV elements:

NLTV Resistor:

- Use the rheostat with a moving terminal to explain passive TV resistor;
- Leads nicely to concepts such as minimal power consumption, minimal energy depletion in any fixed time period etc.
- Extend concept to non-passive;
- Separable (in terms of time and voltage or current effects), non-separable; many to one mappings etc.

NLTV Capacitors:

- Conceptually a storage element with one moving plate;

- Distance between plates cannot be negative or infinity;
- Remains a capacitor wherever the plate was positioned;
- Energy storage function which is a function of the state variable (q or v) and position of the moving plate (which is function of time) and its properties; gives practical meaning to time-varying behavior;
- Start with the energy function and define other variables like voltage etc.
- As in NLTI case, q, the charge being the independent variable offers the more general waveform;
- Connection to time-varying locally positive definite functions, decrescent functions, radially unfounded functions.

NLTV Inductors: Use duality

NLTV transformers and Gyrators: Similar to NLTI case

m-port NLTV mutual inductors and capacitors:

Interconnections / restrictions:

- KVL and KCL laws still hold good;
- Nodes with only Cs (or mutual Cs) or loops with only Ls (or mutual Ls) not allowed from a practical perspective (one state variable gets constrained by other state variable values – goes against the definition of a state variable);
- Use of static elements (such as resistors) should be governed by the way they are interconnected with reactive elements– a current controlled R (only) if it comes in series with an inductor, for example;
- KCL at a node / KVL for a loop leads to natural state equations in the form:
- $\dot{\mathbf{Q}} = \mathbf{f}_{\text{nodes/elements}}\left[\mathbf{Q}, \Phi, \mathbf{f}_{\text{elements}}[t]\right]$, $\dot{\Phi} = \mathbf{f}_{\text{loops/elements}}\left[\mathbf{Q}, \Phi, \mathbf{f}_{\text{elements}}[t]\right]$ where the dynamics incorporate the elements' I/O mappings and the interconnection (the architecture) information;
- The solution $\mathbf{Q}, \Phi$, and then $\mathbf{V}_c = \mathbf{V}[\mathbf{Q}]$, $\mathbf{I}_L = \mathbf{I}_L[\Phi]$ can be found for any kind of mapping; Thus making charge and flux as independent variables leads to more complex responses

Simple circuits, Properties of NLTV dynamics, Explanation of known results or Generation of new results:

Circuits formed with globally passive elements:

- Dynamics comes out nicely as state equations in terms of charge and the flux variables as mentioned before;
- Connection between relaxation and energy-mimina points of individual storage elements, and stable/unstable/multiple equilibrium points;
- For NLTI / TV systems, why we need to talk of stability of equilibrium points and not that of the system or the dynamics;

- Stored energy = Sum of energy in the various reactive (energy storage) elements = One Lyapunov function (not just a candidate);
- Use of C's and L's alone lead to complex energy functions because of the nonlinear mapping;
- Add mutual L's or C's, you get much more complex (non-separable) functions.
- Derivative of the stored energy function = Sum of power entering the reactive elements = negative of the sum of power going into other (non-storage) elements < 0 if globally passive static elements are used;
- Can also be achieved by both kinds of elements, cancellation etc. but may or may not important from a conservative design perspective;
- Why asymptotic stability (concept similar to response to initial conditions in LTI systems or transient response) doesn't' always mean BIBO or Total stability (response under low amplitude excitation) as is the case in LTI systems;
- Or a globally passive circuit can lead to unbounded response under bounded input conditions – Explained from a simple circuit and elements perspective: $\dot{q}(t) + q(t)/(1+q^2(t)) = i_s(t) = A\sin(\omega t)$

Going from Circuit to Dynamics Vs Dynamics to Circuit:

- In the latter, proper circuit synthesis procedure needs to be used to arrive at the proper circuit or else we may get a circuit (a non-passive circuit for example) that leads to wrong conclusions (The equilibrium point unstable);
- Same as in the search for LFs. Lack of a LF doesn't mean the equilibrium point is unstable;
- Hence, from a design perspective, start with a generic circuit architecture (but with the desired properties) and do the optimization within that framework.
- General forms of dynamics derived from various circuit architectures:
- The general form should include mutual inductors;
- The algebraic equation $\mathbf{A}^t\mathbf{P} + \mathbf{PA} = -\mathbf{Q}$ (for LTI systems stability checking) and the corresponding circuit architecture;
- Implication from circuit synthesis perspective (assume the static, lossy circuit and see if the rest of the circuit is reactive);

Non-passive elements and Circuits:

- Use in explaining known dynamics with limit cycles;
- Easy of arriving at new ones.
- Limit cycles Examples:
  - 1) A LTI lossless circuit with time-varying non-passive resistors. The LTI lossless circuits response trajectory (or one trajectory) coinciding with the trajectory where the static non-passive element transitions from passive to active;
  - Example 2: Limit cycle when they don't coincide;
  - Example 3: The relaxation points of the reactive elements and the transition point for the non-passive elements are different.
- Starting from non-passive circuits may not lead to conclusive evidence (like stability of an equilibrium point), but a passive circuit does.

    - Example: A circuit with two LTI capacitors, a non-passive, NL 2-port static network Vs a circuit with two NLTI capacitors and a 2-port LTI gyrator – leading to the same dynamics;

Applications:

Control:
Nonlinear Dynamic Control

- Error model for the closed loop system Vs Dynamics of a globally passive circuit with the relaxation points at (and only at) the origin;
- Use of the new circuit approach leads to results similar to what is known as integrator back-stepping (a well followed research topic) and much more advanced results easily without all the analytical problems associated with it;

State estimation from partial state observation:

- Same as in nonlinear dynamic control with the added restriction that:
    - The sub-dynamics corresponding to the unknown state variables point to a stable sub-system
    - And the circuit leading to the error dynamics should be such that the estimation dynamics uses only the observed state variables;

Adaptive Control:

- Circuit interpretation of the task of adaptive control;
- Various adaptation dynamics using the building block approach;
- Importance of architecture in adaptation (sensitivity issues)

Signal Processing:
LTI Filtering Vs NLTV Filtering:

- NLTV filtering to overcome time Vs frequency domain dilemma in LTI filtering;
- Use of specific circuit architectures to simplify design task;

Time-Varying / Non-Linear Filtering Vs Kalman Filtering:

- Kalman filtering (a time-varying design where the Kalman gain is modified based on prior knowledge and doesn't make use of error in estimation);
- True nonlinear, time-varying filtering with proper coupling between estimation variables and the gain matrix using the Circuit building block approach;

Chaos Circuits Design:

- Reactive elements with equilibrium points at the origin;
- Non-passive resistors to make the origin unstable;
- Other finite-valued equilibrium points that cannot be stable (since the reactive elements prefer the origin only!);

- Make the resistors passive as the state becomes large to prevent escape to infinity.
- Make mappings discontinuous (or piece-wise linear);
- Use more complex elements in existing chaos circuits / dynamics to produce new ones;

Chaos Synchronization:

- Very similar to state estimation problem;
- The error model dynamics should correspond to a globally passive circuit with a stable equilibrium point at (and only at) the origin;
- The receiver derived from the circuit dynamics should depend only on the state variable available at the receiver.

Fuzzy logic / Fuzzy Control:

- Interpretation of fuzzy logic as a tool for producing nonlinear mappings;
- Such mappings are by nature static. For example, using the error and the derivative of the error, a fuzzy control produces a well defined (no fuzziness here) mapping, $F[e, \dot{e}]$, in general a nonlinear mapping;
- Use of such systems even in LTI systems makes stability analysis tough;
- Use circuit building block concept to make the fuzzy mapping as a part of the building blocks;
- For example, we can associate the mapping to a suitable device such as a transformer;
- The resulting circuit / dynamics will lead to fuzzy control with memory or dynamic / recursive fuzzy control.

Neural Control:

- Similar to fuzzy control