

Intelligent Control of Model Helicopters -A Building Block approach

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Abstract—There is a growing interest in the modeling and control of model helicopters using nonlinear dynamic models and nonlinear control. A helicopter is an underactuated system with fewer independent control actuators than degrees of freedom to be controlled, making the control difficult. Further, most of the models and control are derived for separate flight conditions such as take-off or hovering because of the complexity involved. This simplification in fact seems to make the problem more difficult since the simplified model might not have the properties associated with the original physical system. Analytical techniques based on Lyapunov theory are then used to design the controller and the design can become extremely complex. In this paper, we present a new Nonlinear and Adaptive controller design. The design methodology basically involves making the combined dynamics of the helicopter and the controller to resemble the dynamics of a nonlinear time varying electrical circuit having the required properties, using a process similar to reverse engineering. This conceptually simple procedure leads to a new general class of complex nonlinear and adaptive controller algorithms without the use of or the necessity for complex analytical tools, and hence can be understood and applied easily. Even though the solution provided here is for a particular flight condition, this approach can easily be extended to multi-flight conditions.

I. INTRODUCTION

Analytical techniques based on linearization and Lyapunov theory are commonly used in the design of controllers for model helicopters. Here we look at the complete system consisting of the helicopter and the controller {and in fact all nonlinear time-varying (NLTV) or non-autonomous systems} from a new and interesting perspective and arrive at a easy and elegant approach for the controller design. The new paradigm is shown in Fig. 1. As can be seen from the figure, it is a bottom-up approach for system design, where we start with the electrical elements (or in fact their I/O mappings) necessary to form various kinds of NLTV electrical circuits and proceed to form circuits, and the general form of the dynamics in terms of the I/O mappings of the various elements. Such generic equations from circuits with the appropriate conditions (applicable to the problem being solved) are then used as templates for

further design, a process very similar to reverse engineering. As we will show in this paper, the new approach makes the design of complex NLTV systems and their applications to a number of problems relatively an easy job. In this paper, we apply it to nonlinear control of helicopters, and in another paper in this conference, we discuss the application of the new approach to signal estimation.

Key concepts such as power, energy etc. that are unique to the analog world are used here to define the various NLTV electrical elements and their I/O mappings. Implementation of such devices and circuits in the analog domain is very complicated. Luckily, digital implementation has become the preferred mechanism for most applications, and that is what we propose here. We will derive the required dynamics in the analog domain, and transform it to the digital domain using proper A/D transformations. We should also note that the basic properties such as loss or lossless can be preserved much more easily in the digital domain even when finite precision arithmetic is used. Hence, in our solution, we use the best of the analog and the digital world.

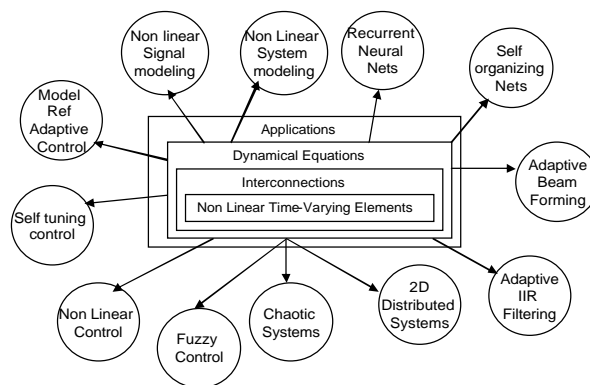


Fig. 1. A Bottom up approach for system design.

The method proposed here is somewhat similar to solutions based on concepts such as passivity, positive real (PR) functions, and dissipative system models (such as Lagrangian and Hamiltonian Systems) that have long been used in areas such as I/O stability and controller design.

However, the previous research uses such concepts from a macro or global perspective whereas we work at a micro or element level. Even though the distinction is very subtle, the new approach provides us the flexibility to tailor the circuit / the dynamics to the application of interest and also eliminates the necessity to deal with analytical tasks such as finding the Lyapunov function etc. The solutions for different applications based on passivity concepts given in a recent book, “Nonlinear control for under-actuated mechanical systems,” [1] illustrate the complexity that we face when we work at the macro level. The solution for helicopter control given here and the estimation problem should demonstrate the superiority of the new approach. Our method is similar to reverse engineering where we try to make a new system by studying an existing product. We learn from electrical circuits (formed on a piece of paper) and use that knowledge to design electrical as well as non-electrical systems for various applications. At the same time, we maintain the internal architecture.

II. NEW METHODOLOGY FOR THE CONTROLLER DESIGN

Let us explain the basic concept in general terms first and discuss the helicopter controller design later. Consider Fig. 1, where we show a closed-loop system consisting of a plant and a dynamic controller. Letting \mathbf{x}_p and \mathbf{c} as respectively the state vectors of the plant and the dynamic controller, \mathbf{u} the control law, $\mathbf{x}_p^*, \mathbf{c}^*$ as the steady state values of the plant and the controller state, we will have the plant dynamics, the controller dynamics, and the control law as shown in the figure.

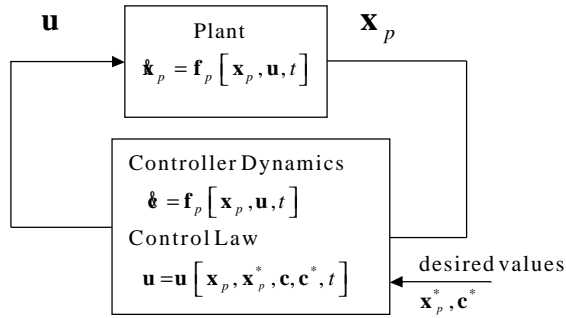


Fig. 2. A block diagram of Dynamic Nonlinear Control.

The controller dynamics and the control law should be selected such that $\mathbf{x}_p \rightarrow \mathbf{x}_p^*$ (and $\mathbf{c} \rightarrow \mathbf{c}^*$) as $t \rightarrow \infty$. And the tracking should be possible regardless of the initial value of the plant state $\mathbf{x}_p(0)$. We can represent the closed-loop system dynamics in the error domain using an augmented error vector $\mathbf{e} = [\mathbf{e}_p; \mathbf{c}_p] = [\mathbf{x}_p - \mathbf{x}_p^*; \mathbf{c} - \mathbf{c}^*]$ as

$\dot{\mathbf{e}} = \mathbf{f}_e[\mathbf{e}, t]$. The control problem is then to find the controller dynamics and the control law (for any given plant) such that the origin is a uniformly asymptotically stable (UAS) equilibrium point of this error dynamics, implying that the error vector will go to the origin as $t \rightarrow \infty$. Whether we can make it globally or only locally UAS would depend upon the plant and the control available. We can look into this UAS requirement from an electrical circuits' perspective. In other words, we can ask what kind of electrical elements would lead to circuits with such a property. Then from knowledge of the general form of the dynamics from such family of circuits and specific knowledge of the plant, we can obtain various control laws and controller dynamics. This in a nutshell is the basis of our work. This approach naturally leads to results similar to the ones obtained using methods such as integrator back stepping, forwarding based on Lyapunov theory etc. But it accomplishes the task without the need for complex analytical manures and also leads to controllers that are much more complex.

Let us consider the basic elements of such a circuit. We can have one-port and multi-port NLTV *passive resistors* (static element or one with no memory), multi-port NLTV *gyrators* and *transformers* (lossless, memory-less elements), one-port NLTV *reactive elements* (*inductors* and *capacitors*), which are lossless elements with memory or energy storage capacity, and multi-port NLTV *mutual inductors*. Since we are defining these elements only in an abstract manner and not really implementing them in analog domain, we can also define multi-port NLTV *mutual capacitors* as the dual of the inductors. Circuits made from such elements will have NLTV dynamics with many equilibrium points, some locally stable and others unstable. To arrive at dynamics which has the origin as the globally UAS equilibrium point, we will restrict the resistors to be globally passive (the voltage to current characteristic should be confined to the first and the third quadrants). Also the reactive elements should have their relaxation points (the stored energy is zero at those points) at the origin and no other relaxation point or points of local minima in terms of stored energy. That is the energy curve should be monotonically increasing and should become unbounded as the independent variables become unbounded. We can easily write down a LF for the dynamics as the sum of energy stored in various reactive elements. This total energy function resulting from the use of only inductors and or capacitors will be a separable function of the state variables. The use of mutual inductors will lead to non-separable energy functions. The derivative of the LF along the system trajectory will be the negative of the sum of power consumed by the passive resistors, which will always be negative for globally passive resistors. We will define just two elements, A NLTV gyrator and a NLTV transformer, here. Few others are defined in the other paper.

NLTV gyrator: A *NLTV gyrator* is a static M-port ($M \geq 2$) element with the I/O relationship (using vector notation) $\mathbf{v}[t, \mathbf{b}] = \mathbf{R}[\mathbf{x}, \mathbf{b}]\mathbf{i}(t)$ where $\mathbf{R}[\mathbf{x}, \mathbf{\beta}]$ is the resistive matrix

of gyrator and is a function of time as well as the state variables and $\mathbf{R}[\mathbf{x},\beta]+\mathbf{R}^t[\mathbf{x},\beta]=\mathbf{0}$. That makes the power flowing into the gyrator zero all the time. That is, a NLTV gyrator is a lossless multi-port device with a skew-symmetric resistor matrix $\mathbf{R}[\mathbf{x},\beta]$. Once again by proper choice of the matrix elements, we can arrive at specific classes of LTV, NLTI, separable, and non-separable models. The elements of the resistor matrix could be selected based on the application and not allowed to become unbounded or indeterminate for finite values of the state. Further, by defining which are the input variables (here i) and which are the output variables and sticking with that definition, we can allow many to one mappings.

NLTV transformer: We can also extend the definition for lossless transformers to NLTV transformers. The I/O relationship is given by:

$$\begin{bmatrix} v_2(t) \\ i_1(t) \end{bmatrix} = \begin{bmatrix} N[\mathbf{x},a(t)] & 0 \\ 0 & -N[\mathbf{x},a(t)] \end{bmatrix} \begin{bmatrix} v_1(t) \\ i_2(t) \end{bmatrix}$$

An ideal transformer is a lossless static device and in this representation, $v_1(t)$ is the input and the load at port # 2 must be a voltage controlled element.

III. HELICOPTER CONTROL USING THE BUILDING BLOCK APPROACH

Let us now discuss the application of the new approach for helicopter control. We use a simplified model of a *Lynx* helicopter [3]. The complete model uses 12 state variables $[x, u, y, w, z, v, q, r, f, q, y]$ where $u = \dot{x}$, $v = \dot{y}$ and $w = \dot{z}$ are the forward, lateral and normal velocities. The angles f, q, y are the roll angle, the pitch angle and the yaw angle. p, q and r are the roll rate, pitch rate and the yaw rate. There are four inputs or control variables u_i $\{i = 1$ to 4 $\}$ associated with the collective, longitudinal cycle and lateral cycle of the main rotor thrust and the torque produced by the tail rotor thrust. This leads to a 12-th order state space and difficult to explain in the limited space. Hence we consider a simplified model with constant lateral velocity $\{y = \dot{y} = \ddot{y} = 0$ and $r(t) = p(t) = 0$ for all $t \geq 0$ $\}$ leading to $u_3 = u_4 = 0$ and a 6-th order model given by:

$$\begin{aligned} \dot{x} &= u; \quad \dot{u} = [-u_1 \sin q - u_2 \cos q u_2]/M \\ \dot{w} &= w; \quad \dot{w} = g + [-u_1 \cos q + u_2 \sin q]/M \\ \dot{q} &= q; \quad \dot{q} = L u_2 \end{aligned}$$

Where M is the helicopter mass, g is the gravitational force and $L = l_h/i_{yy}$, l_h the distance between the rotor hub

and the fuselage center of mass and i_{yy} the moment of inertia, all constants. Let the given desired trajectory be $x^*(t), y^*(t), q^*(t)$ (with smoothly varying higher order derivatives) and \mathbf{u}^* be the corresponding input vector obtained by solving the above state equation. Let $\mathbf{x}_e = [x_e, u_e, y_e, w_e, q_e, q_e]^t$ be the state vector expressed in the error domain. For nonlinear static control, we need to find $\mathbf{u} = \mathbf{u}^* + \mathbf{u}_e$ with $\mathbf{u}_e = \mathbf{u}_e[\mathbf{x}, \mathbf{x}_e]$ such that the error dynamics comes out as $\mathbf{C}\dot{\mathbf{x}}_e = \mathbf{f}_{x_e}[\mathbf{x}_e]$ (where \mathbf{C} is a PD diagonal matrix) corresponding to that of a passive NLTV circuit. \mathbf{u}^* can be found from the 3 equations relating to $\dot{x}, \dot{w}, \dot{q}$ by letting all the variables as the steady state values (replace variables with '*'). We can use a matrix notation involving only these three equations to find \mathbf{u}^* and \mathbf{u}_e . Letting

$$\mathbf{A}^* = \begin{bmatrix} -\cos q^* & \sin q^* \\ -\sin q^* & -\cos q^* \\ 0 & 1 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} -\cos q & \sin q \\ -\sin q & -\cos q \\ 0 & 1 \end{bmatrix} \& \mathbf{C}^* = \begin{bmatrix} gM \\ 0 \\ 0 \end{bmatrix}$$

\mathbf{u}^* can then be written in terms of $\dot{\mathbf{x}}^*$ as $\mathbf{u}^* = (\mathbf{A}^{*t} \mathbf{A}^*)^{-1} \mathbf{A}^{*t} \dot{\mathbf{x}}^* - (\mathbf{A}^{*t} \mathbf{A}^*)^{-1} \mathbf{A}^{*t} \mathbf{C}^*$. By using this value of \mathbf{u}^* in the plant dynamics, results in value of \mathbf{x}_e given as $\mathbf{x}_e = \mathbf{A} \mathbf{u}_e + [\mathbf{A}(\mathbf{A}^{*t} \mathbf{A}^*)^{-1} \mathbf{A}^{*t} - \mathbf{I}](\dot{\mathbf{x}}^* - \mathbf{C}^*)$, where \mathbf{I} is the Identity matrix

For convenience, we split \mathbf{u}_e as $\mathbf{u}_e = \mathbf{u}_{e1} + \mathbf{u}_{er}$ where \mathbf{u}_{e1} is chosen to cancel the $[\mathbf{A}(\mathbf{A}^{*t} \mathbf{A}^*)^{-1} \mathbf{A}^{*t} - \mathbf{I}](\dot{\mathbf{x}}^* - \mathbf{C}^*)$ term. We are hence left with $\mathbf{x}_e = \mathbf{A} \mathbf{u}_{er}$.

Choosing $\mathbf{u}_{er} = [u_{er1}, u_{er2}] = \left[\sum_{i=1}^6 a_i, \sum_{i=1}^6 b_i \right]$, where all

a_i and b_i are functions of q , allows us to write the plant dynamics as $\mathbf{C}\dot{\mathbf{x}}_e = \mathbf{A}[\mathbf{x}, a_i, b_i] \mathbf{x}_e$, where $\mathbf{A}[\mathbf{x}, a_i, b_i]$ is now the 6x6 TV matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -b_1 \cos q & -b_2 \cos q & -b_3 \cos q & -b_4 \cos q & -b_5 \cos q & -b_6 \cos q \\ -a_1 \sin q & -a_2 \sin q & -a_3 \sin q & -a_4 \sin q & -a_5 \sin q & -a_6 \sin q \\ 0 & 0 & 0 & 1 & 0 & 0 \\ b_1 \sin q & b_2 \sin q & b_3 \sin q & b_4 \sin q & b_5 \sin q & b_6 \sin q \\ -a_1 \cos q & -a_2 \cos q & -a_3 \cos q & -a_4 \cos q & -a_5 \cos q & -a_6 \cos q \\ 0 & 0 & 0 & 0 & 0 & 1 \\ b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{bmatrix}$$

This kind of dynamics corresponds to one class of circuits having the required property. We can then let the state vector \mathbf{x}_e to correspond to the voltage across 6 LTI capacitors and

then $\mathbf{C}\dot{\mathbf{x}}_e$ will be the current through those capacitors. Now, for this problem, we need to choose the parameters a_i, b_i such the TV matrix $\mathbf{A}_e[\mathbf{x}, a_i, b_i] = -(\mathbf{R}_R[\mathbf{x}] + \mathbf{R}_g[\mathbf{x}])$, where $\mathbf{R}_R[\mathbf{x}]$ corresponds to the resistance matrix of a 6-port TV passive resistor network (TV PD matrix) and $\mathbf{R}_g[\mathbf{x}]$ is skew symmetric (corresponding to that of a 6-port lossless NL gyrator). Here we have chosen $a_1 = k_1 \sin q$, $a_2 = k_2 \sin q$, $a_3 = -k_3 \cos q$, $a_4 = -k_4 \cos q$, $a_5 = -k_5 \sin q$, $a_6 = 0$, $b_1 = k_1 \cos q$, $b_2 = k_2 \cos q$, $b_3 = -k_3 \sin q$, $b_4 = -k_4 \sin q$, $b_5 = -k_5 \cos q$ and $b_6 = -k_1 k_2$, with $k_i > 0$.

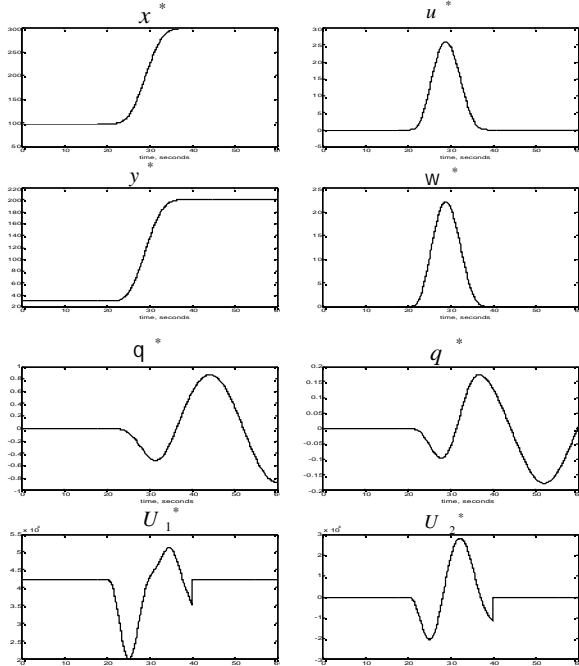


Fig 3 : Graphs showing the desired trajectory of the different state variables

In Fig.3, we show the desired trajectory selected for simulation and the corresponding nominal or desired input values \mathbf{u}^* . In Fig. 4, we show the closed-loop response when there is some error in the initial state. As can be seen from the figure, the nonlinear controller does an excellent job of controlling the helicopter and puts the helicopter in the required trajectory very quickly even though the parameters were chosen almost randomly. We can obtain many other controllers with little effort. For example since the term $\sin q$ appears in the state equations, we can let $\sin q_e$ (rather than q_e itself) be a term in the vector that appear on the RHS of the state error dynamics and choose a new control law $\mathbf{u}_e = [u_{1e}; u_{2e}]$ as before. We can justify this control law by

letting $\sin q_e$ as the voltage mapping of one of the capacitors with the charge equal to q_e . That is, we introduce a nonlinear capacitor in the equivalent circuit

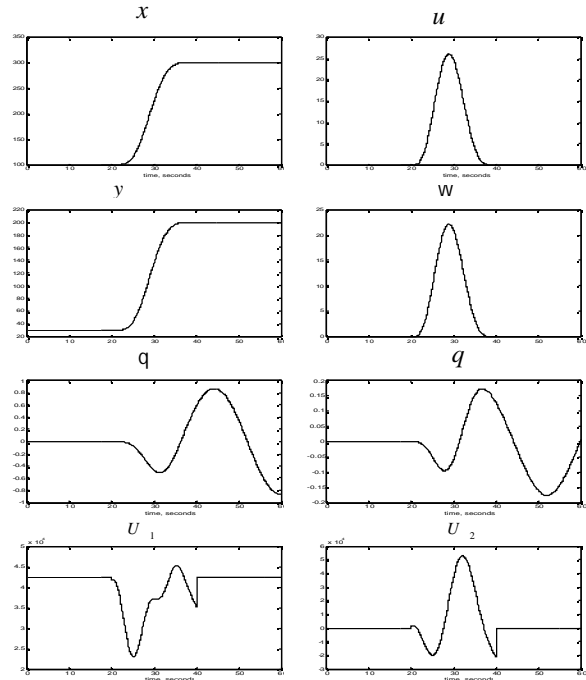


Fig 4 : Graphs showing the actual plant response of the different state variables

IV. SUMMARY

In this paper, we presented a new Nonlinear and Adaptive controller design for a model Helicopter. The design methodology basically involved making the combined dynamics of the helicopter and the controller to resemble the dynamics of a nonlinear time varying electrical circuit having the required properties by using a process similar to reverse engineering.

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