Nonlinear Signal Processing Vs. Kalman Filtering

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Abstract - Most of the research activities in circuits and signal processing, whether analog or digital, are confined to linear signal processing (LSP). The interest in LSP is due largely to its mathematical tractability and ease of implementation, characteristics that were necessary for the technological climate of the earlier decades. However, LSP is limited in its usefulness and the technology for implementing algorithms and analyzing, modeling and design of complex systems has progressed tremendously. Further, we need to understand and utilize nonlinear signal processing (NLSP) before we can move to large-scale self-organizing or self-learning systems. In our work we have developed a unique and powerful paradigm or methodology for the design of nonlinear dynamical systems and signal processing algorithms. In this paper, we will discuss some aspects of this approach and show results of application of this approach in signal estimation. We also compare the obtained results with that of Kalman filtering, which is essentially filtering using a filter with a time-varying coefficient (Kalman gain). The results indicate that the new NLSP approach is superior and more robust as compared to Kalman filtering.

I. INTRODUCTION

Signal processing, which can be considered as a subset of intelligent information processing is stuck primarily at the simple level of linear processing, [1], [2]. However intelligent information processing is by nature nonlinear, and time varying (in terms of memory, application of rules and learning capability), and there are clear indications that the nonlinear/time varying processing account for most of the "intelligent" results outcome, [3], [4]. Thus, the need arises for systematic approaches for the design of nonlinear and time-varying signal processors. This paper is concerned with the first, that of designing stable nonlinear signal processing systems.

The analytical approach, that is defining proper models for nonlinear differential equations (NLDE) and analyzing the resulting models is the most commonly used technique in many areas, and NLSP is no exception to this trend. However, this approach has not been very successful in NLSP since it is known that even simple first order NLDE can lead to chaotic signals - a situation good to produce nice plots etc, but of not much use in design of stable systems. In our research, which is in its early stages, we have adopted what one may term as an engineering approach or building block approach. Under this approach, we have used the concept of passivity to define a number of elements as building blocks for passive nonlinear electrical circuits. Complex passive nonlinear networks can then be formed by proper interconnection of these various nonlinear elements. When energy storing elements are present in such a network (which themselves can be linear or nonlinear), we can obtain a set of input/output relationships as nonlinear differential equations. The basic property that the network is lossy (consumes energy) ensures that the nonlinear differential equations obtained from the networks would represent absolutely stable systems, and this property holds as long as the individual element values are maintained in their permissible range of values. Thus, to design complex nonlinear systems (a nonlinear signal processor for tracking, for example) and self-organizing systems, one simply has to force the dynamics of those systems to mimic the dynamics of a properly constructed passive nonlinear network, a process akin to reverse engineering.

In our research we have developed the basis for the above approach and applied it with relative ease to a number of problems leading to encouraging results. The fruits of such an approach seems to be endless. For example, the approach can be applied to NLSP, linear and nonlinear controller design (for linear and nonlinear plants), self tuning controllers, model reference adaptive controllers, self-organizing networks, adaptive IIR filter design, adaptive beam-forming, two-dimensional systems, fuzzy systems etc. In this paper we provide some details of this approach and show results of application of this approach in signal estimation. We also show results comparing our approach to Kalman filtering, [5], and its superiority.

II. PROPOSED METHOD

Our approach for nonlinear signal processor design (which is equally applicable to a number of problem domains in the nonlinear systems arena) is based on an entirely new and interesting paradigm. It may be called an "engineering" or a "building block" approach for NLSP design as opposed to the analytical/mathematical point of view adopted by earlier researchers. An engineering or physically motivated approach tries to take into consideration physical properties and constraints based on physical properties at every stage of the design. Passivity formulation (to be defined shortly) is used here to obtain the necessary building blocks for a nonlinear system design. These building blocks can then be interconnected in a proper manner to obtain the general structure for a nonlinear system. To design any system, we can use this general structure and vary the parameters so as to obtain the desired properties. To obtain a digital nonlinear system, we can extract the nonlinear differential equations from the general structure and use a forward difference operator.

Passivity is a term commonly used in electrical network theory to indicate consumption of energy. A passive electrical element (linear or nonlinear) is one which always consumes power/energy (lossy) or at most, consumes no power/energy (lossless). They can be non-dynamic (no memory/can't store energy) or dynamic (stores energy and gives it back at some other time). They can be two-terminal (one-port) elements or multi-terminal (multi-port) devices. A passive linear/nonlinear network is simply an electrical network formed by proper interconnection of various passive linear/nonlinear elements. The interconnections must be such that the basic circuit laws are obeyed. An important property of such networks is that they are stable and remain so as long as the values of individual elements remain in the permissive range for passivity. Thus, if we have proper passive nonlinear elements, we can form stable nonlinear networks, obtain dynamical equations describing such networks in terms of the element parameters and use them as target equations for the proposed nonlinear system.

The above approach assumes that proper circuit elements exist. Only few such elements are available in the open literature. One such element is a passive resistor with a current-voltage relationship given by:

$$i_R(t) = G\tan^{-1}(v_R(t)); \qquad G>0 \tag{1}$$

The power p(t) consumed by this element is given by:

$$p(t) = i_R(t)\, v_R(t) \tag{2}$$

and is always non-negative. In general, the v-i characteristic of a general nonlinear passive resistor has to be confined to the first and third quadrants in the v-i plane and has to pass through the origin.

Nonlinear dynamical elements (capacitors and inductors) and their characteristics have been defined. However, the use of nonlinearity in dynamical elements in passive networks may lead to chaos under external excitation. Hence, we use only linear dynamical elements in our design.

Other nonlinear elements do not exist in the literature. For this purpose, we have defined/invented a number of other passive nonlinear devices[1]. One such device is a nonlinear transformer, a two-port element and has the transfer characteristics given by:

$$\begin{vmatrix} v_2(t) \\ i_2(t) \end{vmatrix} = \begin{vmatrix} N(\,) & 0 \\ 0 & -\dfrac{1}{N(\,)} \end{vmatrix} \begin{vmatrix} v_1(t) \\ i_1(t) \end{vmatrix} \tag{3}$$

where N( ) is a nonlinear function of the current(s) and voltage(s) in an electrical network in which the transformer is embedded. It should be noted that:

$$v_1(t)\, i_1(t) + v_2(t)\, i_2(t) = 0 \tag{4}$$

regardless of what ever form N( ) takes[2]. Thus, an ideal nonlinear transformer is a lossless, non-dynamic (or memoryless) two-port device. Though an analog implementation of this device (and other devices invented) is feasible, we will be using digital implementation for the NLSP application. The digital implementation would allow us to realize such elements/devices with ideal characteristic as defined.

Another device that we have invented, and that is highly useful, is the multiport nonlinear gyrator. If we denote V and I as:

$$V = [v_1, v_2, \ldots, v_N]^T \tag{5}$$

$$I = [i_1, i_2, \ldots, i_N]^T \tag{6}$$

where $v_1, \ldots, v_N$ are the voltages across the gyrator ports, and $i_1, \ldots, i_N$ the currents into the ports of the gyrator, we have the relationship between the voltages and currents as follows:

$$I = Y\, V \tag{7}$$

[1] Patent applications are being submitted for some elements referred in this paper

[2] We can place constraints based on the nature of each element. For the transformer, we may require N( ) to be positive at any time instant and a continuous function of time.

Here,

$$Y = [\, y_{ij}(\,)\,], \qquad i,j = 1,\ldots,N \tag{8}$$

is the admittance matrix of the nonlinear gyrator and satisfies the constraint:

$$Y + Y^T = 0 \tag{9}$$

The elements $y_{ij}(\,)$ can be complex functions of the current(s) and voltage(s) in an electrical network. As an example, for a two port nonlinear gyrator, we may have:

$$\begin{vmatrix} i_1(t) \\ i_2(t) \end{vmatrix} = \begin{vmatrix} 0 & v_1^2 - v_2 \\ v_2 - v_1^2 & 0 \end{vmatrix} \begin{vmatrix} v_1(t) \\ i_1(t) \end{vmatrix} \tag{10}$$

We can show that:

$$I^T V = 0 \tag{11}$$

for any functions $y_{ij}(\,)$. That is, a nonlinear gyrator is a lossless and non-dynamic multiport device.

We can connect these devices to form a passive nonlinear electrical network. The number and type of elements chosen will depend upon the system to be realized. For example, a network with 4 state variables can be realized as shown in Fig. 1.

We have used this particular architecture (gyrator, nonlinear resistors and linear capacitors) in many of our applications as it is known in linear passive network theory that any kind of linear systems with complex poles can be realized using linear gyrator/capacitor combinations.

The dynamic equations of this network can be written in the state-space form as:

$$C\dot{V} = YV - F(V) + I \tag{12}$$

where C is a diagonal matrix with the capacitor values, V is the vector of voltages, $\dot{V}$ is the vector of derivatives of voltages, F(V) the

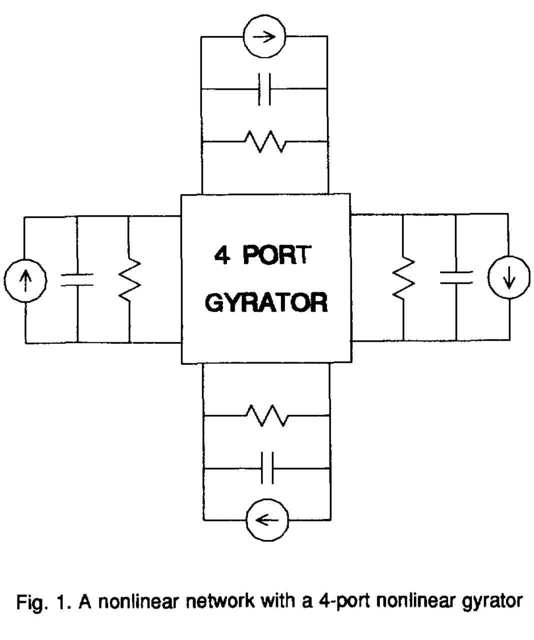

Fig. 1. A nonlinear network with a 4-port nonlinear gyrator

vector of currents through the nonlinear resistors, Y the admittance matrix of the nonlinear gyrator, and I vector of source currents. The set of equations in (12) represent a stable network or system as long as the element values are in the permissible range so as to retain the lossy or lossless property. The stability property holds good even if the terms in the various nonlinear elements are defined by complex functions and the network is highly complex.

Now, to design any stable system, we simply force the system dynamics to take the form given in equation (12). This concept can be applied with relative ease to design stable controllers, stable estimators, etc.

III. APPLICATION TO SIGNAL ESTIMATION

We can describe our approach for signal estimation and the difference from Kalman filtering using, block diagrams of both processes, shown in Fig. 2 and 3. As can be noted from Fig. 2, a Kalman filter is more of a time-varying filter in which K(t), the Kalman gain, is updated continuously and used in estimating the signal. However, the available estimated signal is not used in turn in the updating of K(t). On the other hand, the new approach leads to a truly nonlinear filter where there is high degree of coupling between the gain update block and the signal estimator, as shown in Fig. 3. The selection of the degree of coupling, various parameters/terms in the nonlinear filter etc., forms a part of the design procedure for the new approach and will be described in another paper. Though the circuit is highly nonlinear, since the dynamics of the complete system would be chosen so as to mimic the dynamics of a passive nonlinear network, we are guaranteed of the stability. In our simulations, we found the nonlinear filter to be more effective and highly stable as compared to the Kalman filter.

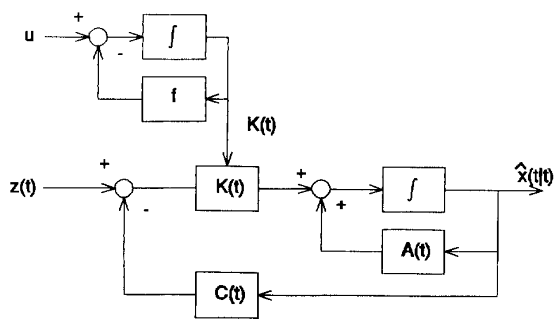

Fig. 2. A block diagram of signal estimator using Kalman filter

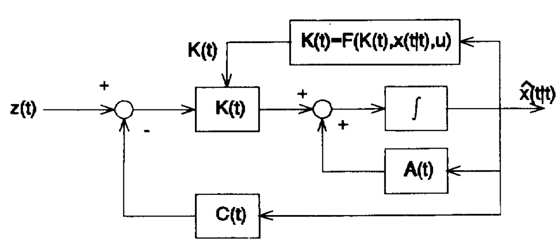

Fig. 3. A block diagram of signal estimator using passive network approach

Let us consider the details for Fig. 3, the nonlinear filter architecture. For the sake of clarity, we will use a very simple model to illustrate the concept. Considering the problem of signal estimation, let the message and the observation models be as follows, [6]:

$$\dot{x}(t) = -x(t) + w(t) \tag{13}$$

$$z(t) = \frac{1}{2}x(t) + v(t) \tag{14}$$

$$x(0) = 1 \tag{15}$$

$$E\{w(t)w(\tau)\} = 6\delta(t-\tau) \tag{16}$$

$$E v(t) v(\tau) = \frac{1}{2}\delta(t-\tau) \tag{17}$$

$$E\{w(t)\} = E\{v(t)\} = E\{w(t)v(\tau)\} = 0 \tag{18}$$

The classical Kalman filter estimator equations for this problem can be represented in the form of the matrix equation as follows:

$$\begin{vmatrix} \dot{\hat{x}}(t\,|\,t) \\ \dot{V}_x \end{vmatrix} = \begin{vmatrix} 0 & (z - \frac{1}{2}\hat{x}) \\ 0 & 0 \end{vmatrix} \begin{vmatrix} \hat{x} \\ V_x \end{vmatrix} - \begin{vmatrix} \hat{x} \\ f(V_x) \end{vmatrix} + \begin{vmatrix} 0 \\ u \end{vmatrix} \tag{19}$$

where u equals 6, and:

$$f(V_x) = 2V_x + \frac{1}{2}V_x^2 \tag{20}$$

Now, let us show how our approach can be applied to the same problem. We choose the expression for the estimator to comply with equation (12). Then, we add the term -(z-x/2) to the estimator (19) to obtain antisymmetric matrix Y and choose

$$f(V_x) = 2V_x + \frac{1}{2}V_x^3 \tag{21}$$

as a nonlinear function of V_x and representing the current through a passive resistor. It has to be noted here that at this point, we have not used any optimization procedure or an exhaustive search for the selection of the various terms. These and other issues are under investigation now. Finally, the source u is selected to force V_x to 2 as x approaches 2z, leading the estimation/Kalman gain update expressions:

$$\begin{vmatrix} \dot{\hat{x}}(t|t) \\ \dot{V}_x \end{vmatrix} = \begin{vmatrix} 0 & (z-\frac{1}{2}\hat{x}) \\ -(z-\frac{1}{2}\hat{x}) & 0 \end{vmatrix} \begin{vmatrix} \hat{x} \\ V_x \end{vmatrix} - \begin{vmatrix} \hat{x} \\ 2V_x+\frac{1}{2}V_x^3 \end{vmatrix} + \begin{vmatrix} 0 \\ u \end{vmatrix} \tag{22}$$

The corresponding differential equation for this problem is:

$$\frac{dx}{dt} = -x \tag{23}$$

and the solution for the initial condition x(0) = 1, is:

$$x = e^{-t} \tag{24}$$

Estimators described by equations (19) and (22) were simulated, and tested with several input signals.

We first tried the estimators with signals of the form:

$$x(t) = e^{-t} + N \tag{25}$$

where N is Gaussian noise. In all the examples, we show the first 500 points of estimation with a sampling time of 0.01. In that case, both estimators, described with equations (19) and (22) showed identical results, and that is illustrated in Fig. 4 and 5. In Fig. 4, we have the input signal of the form (25), with standard deviation of the noise 0.01, and in Fig. 5, the absolute errors of estimations with both estimators. It can be noted that both estimators provide the same results.

An important characteristic of estimators is their robustness to initial conditions. Insignificant dependance on initial conditions is very desirable since the initial conditions in many cases represent no more than an educated guess on the part of the designer. For that reason, we tested estimators with the signals of type:

$$x(t) = Ae^{-t} + N \tag{26}$$

In this case, for values A < 1 , both estimators performed identically, but for values A > 1, estimator described by equation (22) performed much better. This is illustrated in Fig. 6 and 7. In Fig. 6 we have input signal of the form (26), with A equal 100, and with standard deviation of the corrupting noise equal to 2. In Fig. 7 we have absolute errors of estimations with estimator (19) (curve 1), and with the estimator from (22) (curve 2). From these figures it is clear that the estimator (22) performs superiorly, ie. it converges much faster, and the error approaches very small values much quicker than with the classical estimator of the form (19). The estimator of the form (22) performed better or equal for all values of A, and for all reasonable signal to noise ratios.

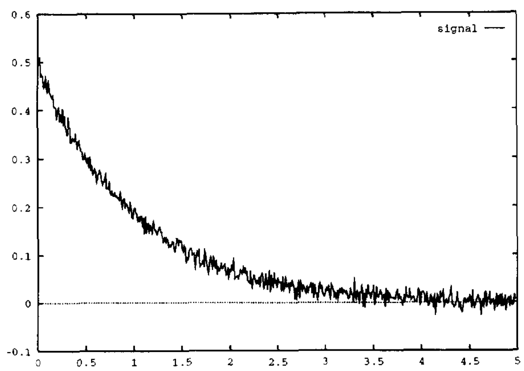

Fig. 4. Input signal of the form (25)

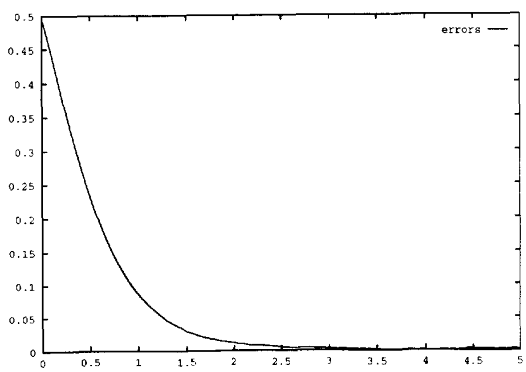

Fig. 5. Absolute errors of estimations with estimators (19) and (22)

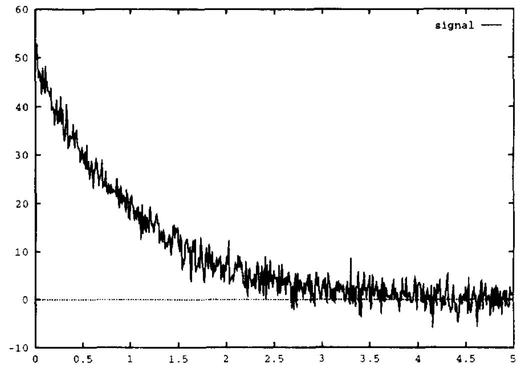

Fig. 6. Input signal of the form (26) with A=100

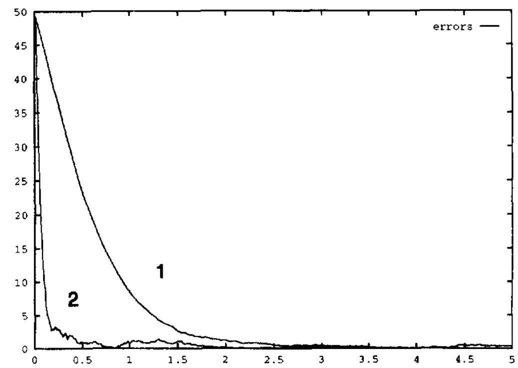

Fig. 7. Absolute errors of estimations with estimator (19), curve 1, and (22), curve 2

IV. CONCLUSION

A new procedure for designing nonlinear signal processors is presented and applied to signal estimation problems. From the results, we can conclude that the new estimator has a superior performance as compared with Kalman filter. The new concept can be applied to other estimation problems, control theory and related problems.

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