with the aid of (3), and from (12) one obtains

$$\mathcal{X}_d(m, n) = \begin{cases} \mathcal{X}_d(m, n) + k & \text{if } m = -1, \ n = 0 \\ \mathcal{X}_d(m, n) + l & \text{if } m = 0, \ n = -1 \\ \mathcal{X}_d(m, n) & \text{otherwise.} \end{cases}$$

(13)

It is observed that the differential cepstrum is invariant with time delay of the signal except at $m = -1, n = 0$ and $m = 0, n = -1$. The additional quantities at $m = -1, n = 0$ and $m = 0, n = -1$ are the time delay measures in $m$ and $n$ directions, respectively. This property can be advantageously used to measure the time delay.

3) Relation between the signal and its differential cepstrum: From (2), one obtains

$$\mathcal{X}_d(z_1, z_2) = X(z_1, z_2) - \frac{\partial Z(z_1, z_2)}{\partial z_1} + \frac{\partial X(z_1, z_2)}{\partial z_2}.$$  

(14)

Using identities similar to (9), we have from (14)

$$\sum_k \mathcal{X}_d(k, 1) x(m - k, n - 1) = (m + 1) x(m + 1, n) + (n + 1) x(m, n + 1).$$  

(15)

The relation (15) is useful for computation of $\mathcal{X}_d(m, n)$ recursively from $x(m, n)$. One limitation to use this relationship to evaluate stable transfer functions even within the very restricted class of filters mentioned.

4) Relation between cepstrum and differential cepstrum: From the definition of the differential cepstrum, we have

$$\mathcal{X}_d(z_1, z_2) = \frac{\partial X(z_1, z_2)}{\partial z_1} + \frac{\partial X(z_1, z_2)}{\partial z_2}.$$  

(16)

Taking the inverse $2$-D $z$-transform using identities similar to (9), one obtains

$$\mathcal{X}_d(m, n) = (m + 1) \mathcal{X}(m + 1, n) + (n + 1) \mathcal{X}(m, n + 1).$$  

(17)

This relation can be effectively used to evaluate the cepstrum from the differential cepstrum or vice versa.

V. CONCLUSION

The 2-D differential cepstrum is discussed as a new basis for homomorphic deconvolution. It is well defined and does not require any phase unwrapping algorithm as in the case of the cepstrum. It has some useful properties such as scale and shift invariance.

The computation of a stable 2-D differential cepstrum by a recursive method is possible for minimum phase signals. One can easily evaluate $\mathcal{X}_d(m, n)$ using the DFT (any FFT algorithm) provided that the DFT of $x(m, n)$ possesses nonzero finite values.

Other applications of the differential cepstrum or vice versa.

REFERENCES


where "*" denotes complex conjugate. Thus a 2nd-order polynomial in \( s \) leads to a 2nd-order polynomial in \( s_1, s_2 \).

The author claims that the conditions derived in the paper lead to all possible stable filters of the type discussed there. In the next section, we give an example of a filter belonging to that group, but which cannot be obtained using the method presented there.

II. COUNTEREXAMPLE

For a 1\text{D} 2nd-order polynomial

\[ D(s) = ds^2 + dz + d_0 \]  

(4)

the 2\text{D} 2nd-order polynomial obtained is given below:

\[ D(s_1, s_2) = \begin{bmatrix} d_0 & d_1 & d_2 \\ d_1 & P_{11} & P_{12} \\ d_2 & P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ s_1 \\ s_2 \end{bmatrix}. \]  

(5)

Letting

\[ d_3 = \sqrt{4 - 2d_1 - \left( \frac{d_2 - d_0}{2} \right)^2}; \]
\[ \alpha = -\left( \frac{d_3}{d_0} \right)^2 + e^{2\alpha}; \]
\[ \beta = \frac{d_1}{d_0} \left( \frac{d_3}{d_0} \tan\theta \right) e^{\alpha}; \]

(6)

it is shown that

\[ P_{11} = 2(\alpha d_0 + d_3) \]
\[ P_{12} = \alpha d_1 + 2d_3 \beta \]
\[ P_{22} = d_0 (\alpha^2 + \beta^2) \]  

(7)

Therefore \( d_2 = 2.1715728, \) \( d_1 = 0.8284272, \) \( d_0 = 1, \) \( d_3 = 1.4142136 \) corresponding to the denominator coefficients of a stable 1\text{D} reference filter; we require that

\[ D(s_1, s_2) = \begin{bmatrix} d_0 & d_1 & d_2 \\ d_1 & P_{11} & P_{12} \\ d_2 & P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 1 \\ s_1 \\ s_2 \end{bmatrix}. \]  

(8)

Now consider the 2\text{D} polynomial given below:

\[ D(s_1, s_2) = \begin{bmatrix} 1 \\ 0.8284275 & 2.1715728 \\ 0.8284275 & 0.125 \end{bmatrix} \begin{bmatrix} 1 \\ 0.8284275 & 2.1715728 \end{bmatrix}. \]  

(9)

This polynomial obviously violates the condition for \( P_{11} \) given in (8) and hence cannot be obtained by the method given in that paper. However, it can be shown that

\[ D(s_1, s_2) \neq 0 \quad \text{for all real } \omega_1 = \text{Im} \{s_1\} \text{ and } \text{Re} \{s_2\} \geq 0. \]

That is, \( D(s_1, s_2) \) can be the denominator polynomial of a stable 2\text{D} transfer function \( G(s_1, s_2) \) satisfying the condition \( G(s_1, s_2) = G(s_2, s_1) \) and \( G(s_1, 0) = G(s_1) \) and still cannot be obtained by the method given there.

III. CONCLUSION

It has been shown that the method by Reinhard Bernstein does not lead to all stable filters, even in a very restricted class of filters as claimed in that paper. More importantly, we would like to point out here that a solution for the design of all stable 2\text{D} recursive filters (both quarter-plane and nonsymmetric half-plane) has already been obtained by the author and has been available for quite some time now [2-4]. Hence any attempt to obtain results on restricted class does not add any to existing results.

REFERENCES


On a Conjecture of J. L. C. Sanz and T. S. Huang

HEINZ-JOSEF SCHLEBUSCH AND WOLFGANG SPLETTSTÖSSER

Abstract—In [1] J. L. C. Sanz and T. S. Huang conjectured that the DFT implementation of the Papoulis-Gerchberg algorithm for the extrapolation of band-limited signals does approach the continuous extrapolation. In this respect we prove a result on the approximation of band-limited functions by trigonometric polynomials, simultaneously increasing its degree and the period-length. This will imply that the above conjecture is indeed true.

I. INTRODUCTION

Let \( g: [-T, T] \to \mathbb{C} \) be a known segment of an \( \Omega \)-band-limited function \( f: \mathbb{R} \to \mathbb{C} \) of finite energy, i.e., \( f^* (\omega) = 0 \), \( \omega \in [-\Omega, \Omega] \), \( f \) being the \( L^2 \)-Fourier (Plancherel) transform of \( f \), and \( g(t) = f(t), t \in [-T, T] \). The (continuous) extrapolation problem then is to find \( f(t), t \in [-T, T] \) in terms of \( g \). This problem is solvable in principle by means of analytic continuation, since the Paley-Wiener theorem implies that \( f \) is the restriction of an entire function of exponential type \( \Omega \) to the real line \( \mathbb{R} \), i.e., \( f \in B_{\Omega,2} \), where \( B_{\Omega,2} \) is the space of all entire functions \( f: \mathbb{C} \to \mathbb{C} \) which satisfy, for some \( K > 0 \), the growth condition

\[ |f(z)| \leq K \cdot e^{\Omega |z|}. \]  

(1)

An iterative procedure converging to this continuation on \( \mathbb{R} \) was...