1. INTRODUCTION

1.1 General Theme

The main thrust of this book is on Nonlinear Dynamical Systems and Adaptive Systems. The basic concepts and the underlying principles are presented from an engineering perspective. The application of nonlinear and adaptive dynamics in nonlinear systems modeling and control, and signal processing are discussed. It is shown that the two emerging areas, Fuzzy Control and Neural Networks, are indeed special classes of nonlinear systems and can be handled through a new paradigm for nonlinear systems and adaptive systems design proposed and developed in this book.

This book, as indicated in the preface, is based on what might be termed an "Engineering and Reverse-Engineering approach" for nonlinear and adaptive/learning dynamical systems as opposed to the analytical/mathematical approach adapted by earlier researchers. The difference in the philosophy is shown in Fig. 1. In Fig. 1a, we depict the traditional approach adapted by earlier researchers. It can be called a top-down approach where we start with the general form of differential equations (nonlinear in this case), incorporate constraints to arrive at special classes of differential equations, impose further conditions such as stability and use certain classes for certain application domains. The triangle with a shrinking base indicates that as we move down, the domain for such entities shrinks or reduces. More importantly, any physical insight that might otherwise be available about the system is rarely used in this approach.

This book advances a bottom-up approach for dealing with nonlinear dynamical systems (Fig. 1.1b). We start with an "element box" (similar to a toolbox) consisting of basic nonlinear electrical elements having the necessary properties, and elements needed to build stable nonlinear electrical circuits. These nonlinear electrical elements can be interconnected to yield various stable electrical circuits and the corresponding nonlinear dynamical equations. Again, there is a certain shrinking of the base, since the interconnection would impose certain restraints (for example, a closed-loop with only dynamical capacitive elements or a node with inductive dynamic elements needs to be avoided). However, the reduction is less as compared to a move from all differential equations to certain classes etc. in the classical approach. This approach, though conceptually simple, leads to a methodology that is simple to understand, yet powerful and applicable to a number of problem domains (Fig. 1.2 shows the areas to which the concept has already been applied and discussed in this book). It is expected that this book would attract many new researchers to this approach and lead to the application of the concept to many other areas.

Under the new approach, a number of basic elements based on power consumption, generation, and storage properties are identified as building blocks for nonlinear and time-varying electrical circuits. Each element is characterized by a mathematical relationship connecting its input and output variables. Complex passive electrical circuits can thus be formed by proper interconnection of these elements. When lossless, energy-storing elements are present in such a circuit (they can be linear, nonlinear, and time varying), we can
obtain a set of input/output relationships as dynamic (state) equations. When the constructed circuit is lossy (consumes energy), the corresponding dynamic equations are asymptotically stable\(^1\). By proper choice of the elements’ characteristics, we can also ensure the More importantly, the stability property holds as long as the individual elements' characteristics are maintained in their permissible range. “Reverse Engineering” techniques where one learns from an existing system or builds systems to mimic an existing system can then be used to design any system (electrical and non-electrical) from such electrical circuit prototypes. Furthermore, elements, which are neither fully lossy nor fully active, can be used to form more complex circuits and dynamics, explain away the properties of systems that are unstable or chaotic, devise methods to control such systems, and build new, exotic systems such as recurrent neural networks.

The building block approach, perhaps new to the electrical engineers trained in the 1970’s and onwards (mini- and micro-computer era), can be found in areas such as civil and mechanical engineering. For example, in building construction, bricks, mortars, 2 by 4’s, nails etc. form the basic building blocks. Similarly, in mechanical engineering, one progress from raw materials to metals to parts to systems to system characterization. Mathematics is primarily used to establish the properties or characteristics of parts/systems that are first-of-all physically possible. A similar approach seems to have existed in electrical engineering during the good old analog-technology days. For example, mathematical concepts such as positive real function etc. were developed to characterize electrical circuits made up of elements that were already available. The minicomputer revolution of the 70’s and later developments seem to have contributed to the move of the electrical engineering from a physical level to an abstract, mathematical level. In this book, we strive to strike a balance between the abstract, mathematical level and the physical level. The benefits from such a balanced treatment would be obvious to the readers.

1.1.1 A simple example to illustrate the general theme

We will now use a simple example from linear, time-invariant (LTI) signal processing\(^2\) to illustrate the general theme. An important task in LTI signal processing is designing filters to extract desired information from a noisy version of the same. Assuming continuous/analog filter realization, the design can be carried out in the frequency domain using the system transfer function:

\[
H(s) = \frac{\text{LT}[y(t)]}{\text{LT}[x(t)]} = \frac{1}{a(N)s^N + a(N-1)s^{N-1} + \ldots + a(1)s + a(0)} = \frac{1}{D(s)}
\]

(1.1)

where \(y(t)\) is the desired (output) signal, \(x(t)\) is the observed, corrupted signal consisting of the desired signal and an additive noise, \(\text{LT}\) stands for Laplace transformation, and ‘s’ is the complex frequency variable associated with the Laplace transformation. Assuming the desired signal to be a low frequency signal, \(H(s)\) can be written in the form of an all-pole transfer function:

\[
H(s) = \frac{1}{a(N)s^N + a(N-1)s^{N-1} + \ldots + a(1)s + a(0)} = \frac{1}{D(s)}
\]

(1.2)

where the value of \(N\), the order of the transfer function, is selected based on the filtering need, and other considerations such as complexity and cost. The design task is to arrive at the values of the denominator coefficients \(a(i), i = 0\) to \(N\), such that the filtering function is accomplished in an optimal manner while maintaining other constraints on the coefficient values. An important constraint in the case of continuous systems is that the resulting system is stable. For LTI systems, the stability requirement is satisfied if and only if

\[
D(s) \neq 0 \quad \text{for Real}[s] \geq 0
\]

(1.3)

That is, the denominator coefficients \(a(i), i = 0\) to \(N\) need to be so constrained that the zeros of the denominator polynomial are in the left side of the s-plane. Thus, the design becomes a constrained optimization problem. A common approach to deal with such a constrained optimization problem (a relatively complex task) is to convert the problem to an equivalent unconstrained optimization problem (relatively, a less complex task) using transformation of variables. For example, the above transfer function can be written as (assuming \(N\) to be even):

\[
H(s) = \frac{1}{a(N)s^N + a(N-1)s^{N-1} + \ldots + a(1)s + a(0)} = \frac{1}{a(N)\prod_{i=1}^{N/2} (s^2 + b_is + b_{2i})} = \frac{1}{a(N)\prod_{i=1}^{N/2} (s^2 + c_is + c_{2i})}
\]

(1.4)

\(^1\) It should be pointed out that the possibilities for the elements’ characteristics are unlimited. Hence care should be exercised in choosing them so that the required property, for example exponential stability, is achieved. The connection to physical devices makes it easier to accomplish than through a pure analytical approach.

\(^2\) Formal definitions for various terms encountered here are given in chapter 2.
where, we have made use of the facts that:
1) any N-th order polynomial with real-coefficients can be written as the product of \( \frac{N}{2} \) second-order polynomials;
2) the coefficients of second-order polynomials with roots having negative real-parts need to be positive; and
3) a transformation of the form:

\[
b_{ji} = e^{c_{ji}}, \quad j = 1,2; \quad i = 1 \text{ to } \frac{N}{2}
\]

(1.5)

...to transform real coefficients to positive coefficients. In this problem, we can use the coefficients as the variables of an unconstrained optimization problem.

Note that the transfer function in equation (1.2) and the corresponding constraint in (1.3) have been arrived from a study of LTI systems that are physically realizable and their specific properties. However, problems arise when we depend exclusively on the use of abstract mathematical models with no concern to physical realizability. For example, the mathematical model of the transfer function in equation (1.2) can be considered as representing the signal flow-graph realization shown in Fig. 1.3a involving three building blocks shown in Fig. 1.3b. This abstract model and the corresponding architecture shown in the figure was not just used as a representation but as a real circuit built using the three building blocks shown (so called analog computers for simulating control systems). Unfortunately, the representation using operational amplifiers, one of the building blocks, and an active device, encompasses not only stable systems but also unstable systems. Hence, an additional constraint, as given in equation (1.3), has to be added. Further, even for stable systems, the architecture corresponding to this abstract model exhibits high sensitivity in its response to coefficient variations. In fact, this later problem simply brought a bad name to the whole area of analog computing and killed any remaining interest in that area.

A different approach proposed in this book is to consider a physical realization in terms of well defined passive (and active, if necessary) elements, and write the transfer function or the input/output mapping in terms of the individual element characteristics. For the example under consideration, an electrical circuit realization of a lowpass transfer function is a doubly terminated ladder architecture as shown in Fig. 1.4. The transfer function \( H(s) \)

\[3\]

in terms of the element values, is given by:

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3 We give the transfer function for a specific case of \( N = 5 \) to avoid the complexity associated with writing a general N-th order transfer function. However, the general form of the transfer function should be obvious from this example.
Thus, instead of using an abstract form of the transfer function as given by equation (1.2), we can use the form in (1.6) and the variables \( c_1, L_2, c_3, \ldots, c_N \) as the variables of optimization with the knowledge that any positive valued set will represent a stable system.\(^4\)

Of course, the above described approach may not be of great value in the design of LTI signal processing systems as we deal with a polynomial in a single variable, \( s \). The task of constraining the roots of this polynomial to the left-hand side of the \( s \)-plane is a relatively easy task. However, the stability issue is not straightforward in the case of other systems, such as nonlinear and multi-dimensional systems, and methods based on the approach described here will be of tremendous value in such situations. Further, an approach based on certain physical models can offer other properties such as reduced sensitivity to coefficient variations and so on. This has already been proven in a number of areas, as we will show in later chapters.

Another important observation that can be made from this example is that a properly constructed electrical circuit is used only in an abstract form. That is, a circuit drawn on a piece of paper with the sole aim of extracting the general form of the input/output relationship in terms of known primitives (element values, here) of the system. By varying the primitives' characteristics within their respective domains, we generate all stable I/O relationships and use them to describe all other electrical/non-electrical systems. Thus, the approach can be considered as taking the art of reverse engineering to a higher plane.

\[ H_s(s) = \frac{1}{c_1L_2c_3L_4c_5Rs^4 + (c_1G_L + c_3)L_2c_1L_5s^4 + (R_1c_2(L_2c_3 + L_4c_5) + c_3L_4G_L + R_1c_2)s^3 + (R_1c_2(L_2c_3 + L_4c_5) + c_3L_4c_5 + L_2c_1c_3 + L_2c_5)s^2 + (R_1c_2 + c_3c_5 + G_L(L_2 + L_4))s + 1 + R_1G_L} \]

where \( G_L = R_L^{-1} \). The condition for physical realizability is that all the element values be positive. Further, it is known that:

1) the transfer functions of circuits made of such elements are stable, and
2) all stable, all-pole transfer functions can be realized using a ladder circuit of Fig. 1.4.

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\(^4\) We may use transformations as given in (1.5) or similar ones to convert the problem to an unconstrained optimization problem.
1.2 Prior Research Vs this work

Nonlinear dynamics is a vast and highly researched area and many excellent contributions have been made by a number of researchers during the last 100 or so years. It is difficult for any author (and especially this author, who managed to pursue this research work with almost no support or resources) to discuss all of them in a single book. This task becomes more complicated given the major goals of this book, namely,

1. An easy and intuitive approach to learning nonlinear dynamical systems;
2. Emphasis on use and design of practical nonlinear systems as opposed to abstract mathematical theory of nonlinear systems;
3. Discussion of various obvious and not so obvious application areas of nonlinear dynamics, and,
4. Selling the new paradigm as an alternate for modeling and designing complex nonlinear systems.

Thus, in this section, we will discuss the earlier research in very general terms, try to reconcile our work with the earlier work, and provide some insights into the differences.

A.M. Lyapunov, a Russian mathematician, did the early seminal work on nonlinear dynamics in the 1890s. Let us consider nonlinear systems described by a set of first-order differential equations:

\[
\begin{align*}
\dot{x}(t) &= f[x(t), t, u(t)] \\
y(t) &= g[x(t), t, u(t)]
\end{align*}
\]  

(1.7)

where \(x(t)\) is the state vector, \(u(t)\) is the input vector, \(y(t)\) is the output vector and \(f[\bullet]\) and \(g[\bullet]\) are vectors of nonlinear functions of the state, the input and the time variable. The original work of Lyapunov assumes that \(u(t) = 0\) and the nonlinear functions \(f[\bullet]\) and \(g[\bullet]\) are not explicitly dependent on the independent variable \(t\). That is, an autonomous nonlinear system model with no external forcing function and its response for a given initial condition is used by Lyapunov to develop the basic concepts of nonlinear systems. The nonlinear functions in the differential equations make even the transient response of such models complicated. Lyapunov developed the various stability concepts (such as asymptotic stability and exponential stability, local stability and global stability) to go with the equilibrium point(s), defined as the value(s) of the state vector \(x\) for which the dynamics in equation 1.7 (with \(u(t) = 0\)) becomes zero. Viewed from a systems’ perspective, the initial condition represents the initial stored energy, the equilibrium point(s) corresponds to a state of zero energy or a local minima of the stored energy, and the response reaching the equilibrium points implies that the initial stored energy some how gets spent. Thus, the true Lyapunov function (LF) is the energy left in the system at any given time and the derivative of the LF along the system trajectory is the net power entering the system. In fact, the development of the Lyapunov stability theory seems to be based on a real-world mechanical system with a nonlinear spring, nonlinear damper and a mass. However, as time progressed, the emphasis shifted away from real-world systems (consisting of real components that are properly connected) and their corresponding models to abstract mathematical models with the resulting complications. Naturally, the abstract models do not point to real-world systems. Thus, instead of a “nice” LF popping out of the system model, we need to go hunting for a LF candidate, and check if it satisfies the conditions for being the LF of that system. If not, we need to remind ourselves that our inability to come up with the proper LF doesn’t necessarily mean the system (or to be precise, the equilibrium point) is unstable. Rather, we need to go back to the drawing table and try again! That is, the Lyapunov stability theorems correspond to sufficient conditions. This remains true even today. Further the nonlinearities present in the dynamics and the use of abstract models exaggerates problems such as observability, measurability and controllability. For example, the way the inputs appear in the models makes it very difficult to arrive at a proper feedback that will stabilize and control the system. Our building block and reverse engineering approach (also known as Synthesis by Analysis in Passive Network Theory), helps to overcome this problem at least for systems of practical interest. We will continue with this issue later.

Theoretically speaking, Lyapunov stability theory can be applied to a study of the response to external excitation through the use of a model in terms of the error state; that is, the difference between the actual state and the expected state. This implies that we have some knowledge about the expected response, which is not always the case. Also, the error model will in general point to a non-autonomous (time-varying) system, much more complex than an autonomous system. Further the problem of finding the LF for the error dynamics remains.

The second approach for analysis of nonlinear systems, known as Input-Output stability, originated with the works of I.W. Sandberg and G. Zames in the early 1960s. This approach uses a system operator model (convolution integral in the case of LTI systems) or mapping from an input space into an output space and uses concepts from functional analysis. The system operator model doesn’t involve the assumption of finite-dimensionality and lumped systems as is the case with the state-space differential equation model and hence...
can easily be applied to distributed systems (ex: system with time-delay) as well. However, circuit theory allows the use of well-defined delay elements as building blocks for passive (and active) systems and hence can be made part of the finite-dimensional state-space models. In fact, circuit elements that lead to multi-dimensional circuits characterized by partial differential equations and the behavior of such circuits have been well researched.

Also, the I/O stability theory doesn’t assume that the system represented by the differential equation is not well posed. That is, there is a unique solution corresponding to each initial condition, and that this solution depends in a continuous manner on the initial state and time. However, it is customary to assume so under Lyapunov stability theory. Thus, it can be argued that the I/O theory is more general than Lyapunov stability theory. On the other hand, we can question if we are really going to build physical systems that are not well posed.

Input-output theory uses the p-th norm, where $p \in [1, \infty]$, given by:

$$\|x\|_p = \left(\int |x(t)|^p \, dt\right)^{\frac{1}{p}}$$

and the corresponding $L_p$ stability for any input (forced response corresponding to “external” stability). On the other hand, Lyapunov stability, which uses energy and power concepts, is based on $L_2$ stability applied to the behavior corresponding to initial conditions (unforced response or “internal” stability). Thus, the two theories appear to complement each other, and I/O theory appears to be more general than Lyapunov stability since it applies for any value of $p$. However, if we assume the nonlinear vectors $f[\bullet]$ and $g[\bullet]$ in the nonlinear system model of (1.7) to be globally Lipschitz continuous, and that the equilibrium point is globally exponentially stable, then the system given by (1.7) is also $L_p$–stable for all $p \in [1, \infty]$. In practice, we will require systems to have exponentially stable equilibrium points, and if there is only one equilibrium point, it is also globally stable. Thus, from a practical perspective both the theories point to similar results.

The I/O stability concepts have been used in feedback control quite extensively and in particular, to a two subsystem configuration as shown in figure 1.5. We can note that the I/O stability of multi-systems and in fact, the present-day control theory centers on the signal flow graph approach (SFG). Unfortunately, we omit the physical constraints as we work at the SFG level.

For example, the SFG of figure 1.5 omits the loading effects, which may not be true in practice.

There are two fundamental theorems in the I/O stability area, the Small-gain theorem, and the passivity theory, applied to the two-subsystem model of Fig. 1.5, and are used quite frequently. According to the small-gain theorem, with certain constraints on the two subsystems $G$ and $H$, the feedback system in Fig. 1.5 is $L_p$–stable if the gain of $H$ is smaller than the reciprocal of the gain of $G$ (or vice versa).

The passivity theorem (applicable for $L_2$–stability only) states that the system of Fig. 1.5 is $L_2$–stable only if the overall dissipation constant of the closed-loop system (which can be thought of as the “effective resistance” of $G$ and $H$) is positive. Or equivalently, a system that dissipates energy is $L_2$–stable. Thus, it can be thought of as an extension to Lyapunov stability results applied to the two-subsystem configuration. Both the small-gain theorem and the passivity theory treat the nonlinear system at a global level. Given the complexities of nonlinear systems, it appears to be prudent to look at nonlinear systems at the element, and interconnection levels (macro level). In this book, we propose such an approach for representation, modeling, design, control, and adaptation of practical nonlinear systems.

Another currently active area of research is in the application of differential-geometric methods for the analysis of nonlinear control systems. Here, techniques from differential geometry such as vector fields, Lie bracket of two vector fields, and various types of Lie derivatives are used to develop nonlinear system analogs of several well known results for linear systems. Some significant results worth mentioning are in areas such as controllability, observability of nonlinear systems, linearization, either locally or globally, a given nonlinear system through techniques such as coordinate transformations, and state feedback or output feedback and disturbance rejection in nonlinear systems. Unfortunately, unlike linear systems, we have almost unlimited choices in nonlinear dynamics in selecting the model, the nonlinearities involved, and the way the inputs appear in the model. Thus, simplified models such as Bilinear System models given by:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{M} u_i(t) B_i x(t)$$

where $u_i, i = 1$ to $M$, are the M inputs and $A, B_1, B_2, \ldots, B_M$ are constant matrices are used in deriving the various results. However, it is not clear if such models correspond to real-world plants. Dynamics from properly connected well-defined electrical elements indicate that they may not, as we will see in this book.

On the other hand, when we work with real-world nonlinear elements / plants and models derived from them, we can make sure that controllability is

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7 There is a SFG representation corresponding to any physical system. However, an arbitrary SFG cannot be converted to a physical system with specific, for example passive, elements.
built in, so to speak, by defining appropriate inputs and the way they are injected into the system. Further, we need to be careful in defining what the outputs are to satisfy the observability criteria. For example, consider the dynamics of a simple first-order passive nonlinear system given by:

$$\dot{x}(t) + f(x(t)) = u(t)$$

$$y(t) = f[x(t)]$$  \hspace{1cm} (1.10)

where $f[\bullet]$ is a many-to-one mapping confined to the first and third quadrants as shown in Fig. 1.6. We can see that the origin is a globally stable equilibrium point. Also, by selecting $u(t) = f_{FB}[x] + u_{ss}$, where $f_{FB}[\bullet]$ is a nonlinear mapping such that the combined mapping $f[\bullet] - f_{FB}[\bullet]$ is monotonically increasing, we can make the system controllable. On the other hand, the many-to-one mapping from the state variable to the output makes this system not observable.

A number of articles on nonlinear dynamics from the nonlinear circuit’s perspective have been written. Worth noting are the results from two publications by J.L. Wyatt, Jr. etc., since power, energy, passivity and connectivity are the central themes of this book. In these two papers, the authors consider various definitions of losslessness and passivity existing in the literature and demonstrate the contradictions between them with several examples. Many of these problems occur due to the complexity that arises due to the use of nonlinearity, as some examples below will demonstrate.

Consider first the definition for passivity for an n-port network based on I/O stability. An n-port network is passive if, whenever the state $x$ at time zero is $0$,

$$\int_0^T \langle v(t), i(t) \rangle dt \geq 0$$  \hspace{1cm} (1.11)

for all admissible pairs $\{v(\cdot), i(\cdot)\}$ and all $T \geq 0$. That is, this definition is based on the zero-state response. However, a simple circuit consisting of just one capacitor with a $v(t)$ vs $q(t)$ characteristic given by $v[q] = q - 1$, as shown in Fig. 1.7a, will be classified as active according to this definition even though it is in fact a passive one. The problem is due to the fact that the zero state $\{q = 0\}$ is not the equilibrium point $\{q_{eq,pr} = 1\}$ of the circuit and energy is indeed delivered by the circuit as the state moves from the zero-state to the equilibrium point. On the other hand, the two port shown in Fig. 1.7b will qualify to be passive according to this definition even though we can extract unlimited energy from this 2-port by connecting an arbitrarily small resistor to the port 2. The conflict here is due to the fact that the 2-port is not completely controllable.

A second passivity definition states that an n-port, storing no energy at $t = 0$, is passive if (1.11) holds for all $T \geq 0$. According to this definition a nonlinear capacitor (Fig. 1.7c) characterized by $v[q] = e^q$ is not passive since it has no...
state of zero voltage (or state of zero stored energy) and it is possible to extract some energy from it for each initial state \( q \).

The anomalies associated with the two passivity definitions can be resolved using the concept of available energy and its bounds. Given an \( n \)-port, let the available energy \( E_A[x] \) be the maximum energy that can be extracted from it when its initial state is \( x \), with the convention that \( E_A[x] = +\infty \) if the available energy is unbounded. Then the \( n \)-port is \textit{passive} if \( E_A[x] \) is finite for each initial state \( x \).

The above passivity definition appears to be most reasonable concept of passivity for lumped nonlinear \( n \)-ports as it does not single out any particular state for special attention, and it does not require that a state of zero stored energy be found. However, the lack of the latter can lead to problems in practical systems. For example, the nonlinear capacitor characterized by

\[
v(q) = q(1 - q^2)e^{(-q^2/2)}
\]

shown in Fig. 1.7d is passive but will lead to unstable systems. This prompted the authors to define a narrower concept of passivity called \textit{stronger passivity}. According to this definition, an \( n \)-port with a state representation \( S \) is \textit{strongly passive} if

1) For each state \( x \) in the state space, \( E_A[x] < +\infty \), and
2) There exists a \textit{relaxed state}, defined as the state where \( E_A[x] = 0 \), in the state space.

It can be shown that an \( n \)-port network with a relaxed state and a completely controllable state representation is strongly passive. Thus from a practical perspective, we need to consider systems which are strongly passive.

![Figure 1-7](https://via.placeholder.com/150)

Figure 1-7. a) A LTI capacitor with a relaxation point at \( q = 1 \); b) A two-port network capable of supplying unlimited energy; c) A nonlinear capacitor whose relaxation point is at \( q = -\infty \); d) Another nonlinear capacitor characteristic with relaxation points at \( q = -\infty \), 0, and \( +\infty \).

To assure stability based on strong passivity consideration, we need to make additional assumptions. For example, consider a circuit consisting of a resistor of 1 \( \Omega \) and a nonlinear capacitor with \( v(q) = q(2 - q^2)e^{(-q^2/2)} \). We can see that the available energy from the capacitor alone is \( E_A[q] = q^2e^{(-q^2/2)} \). That is, the origin is a relaxed state and hence the one-port is strongly passive and the origin is locally asymptotically stable. But we can easily verify that \( q(t) \rightarrow +\infty \) as \( t \rightarrow \infty \) for any \( |q(0)| > \sqrt{2} \). The problem arises here because the capacitor has two more relaxation points at \( q = \pm\infty \), which should be avoided in practical circuits. We should rather expect the stored energy (or storage capacity) go to infinity as \( |q| \rightarrow +\infty \). In fact, for storage elements, we should have \( E_A[x] \rightarrow x^2 \) as \( |x| \rightarrow +\infty \) with \( x \) as the independent variable. Similarly, the resistive elements should consume power at least equal to the amount consumed by some arbitrarily small linear resistor as the independent variable (current or voltage) goes to infinity.

All the above mentioned problems and controversies arise because we try to define global properties such as passivity of general nonlinear \( n \)-ports.
mathematically. On the other hand, nonlinearity is such a general and complex tool that makes such definitions difficult. Thus, at least from a practical perspective, we should concentrate our efforts in defining:
1) the properties at the macro or element level.
2) the constraints on the element characteristics
3) the way the elements should or should not be connected, and
4) the properties of such possible circuits and their potential applications.

Such an approach will take us a long way in realizing the widespread use of nonlinear dynamics in practical applications. In fact, J.C. Willems, a well known researcher in nonlinear control, writes in a recent paper [J.C. Willems, 1997] entitled “On interconnections, Control, and feedback.”

“Models obtained from first principles will seldom be in input-output or input-state-output form, and it is worth asking whether they form a reasonable starting point for the development of a theory which aims at treating physical models. When a physical model is not endowed with a natural signal flow graph, it is asking for difficulties to suggest that it has one.”

Our work takes this comment one step further: “Let us form models based on descriptions of valid components and valid interconnections.”

Our work also differs from the bond graph techniques used for the study of dynamic systems. A bond graph simply consists of subsystems linked together by lines representing power bonds and is used more as an analysis tool. Here, power considerations are used only to define the various elements and the dynamics do not carry the power information explicitly. Further, we use the dynamics that arise from properly connected elements as templates for modeling dynamic systems, making the design of nonlinear systems much easier.

Also, our results differ from that of Lagrange-Hamiltonian equations of motion for mechanical systems which uses a specific model. Here, we form various models from the elements depending on the application at hand.

### 1.3 Materials Covered: An overview

Three important topics are covered in this book: Nonlinear dynamical systems (including self-learning systems), Nonlinear Signal Processing and Nonlinear dynamical control. Each topic itself is of great importance and complex enough to be dealt within separate books. However, we have decided to deal with them together here to give an elementary and broad view of the areas in a unified manner. Readers interested in more detailed knowledge of the materials may use this book as a springboard to jump to other books.

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*The term nonlinear signal processing is used to imply signal processing by nonlinear operators.*

Processing of signals to extract information of interest, the operation of a system, and control of a system can all be represented by a black box as shown in Fig. 1.8a. Thus, we have a physical entity that has as inputs N signals and as outputs M signals, where the output signals depend on the input signals by the transformations or processing effected by the system. The transformation or processing can take a number of forms and in fact, evolved from a simple level to more complex levels as shown in Fig. 1.8b to 1.8f. Fig. 1.8b depicts the case of linear, static processing (LSP), the simplest form of information processing. In this case, it can be noted that the processor takes in the input(s) values at a particular time instant and multiplies the inputs by pre-determined constant values to produce the output(s). Thus, the processing is linear, time-invariant and static or memoryless. That is, the output values depend on the present values only and not the previous values of the inputs.

Fig. 1.8c, shows the next level (in terms of increasing complexity and utility) of processing, that of linear, dynamic processing (LDP). Here, the outputs depend on not only the present input values but also the past values of the inputs. It should be observed that LSP & LDP leads to linear models and the mapping from the input(s) to the output(s) represents a hyper-surface, a very restricted case of mapping. This kind of mapping has inherent limitations; from a systems point of view, a smaller bandwidth (to reduce the impact of unwanted signals or disturbances) leads to sluggish response in the time-domain and so on. For example, if we assume a plant represented by a linear first-order differential equation and controlled by a proportional control, the closed loop system model will be given by:

$$H_{Ls}(s) = \frac{k}{1+s\tau}$$

where the parameter $\tau$ affects both the bandwidth (inversely) and the rise time (proportionately). Thus, we are forced to make a compromise in choosing that parameter value.

Fig. 1.8d depicts the case of nonlinear, static processing (NLSP). Controllers and classifiers based on fuzzy expert systems (FES) and multi-layer feedforward neural networks (MLFFNN) using only present inputs fall under this category. The fact that the mapping resulting from NLSP can be more general than the mappings possible with LSP and LDP indicates that NLSP can provide better performance than LSP and LDP. However, NLSP leads to nonlinear dynamical systems (when used in a closed-loop system as in control) whose stability may not be provable mathematically. For example, in classical nonlinear control theory, we have the famous Aizerman's conjecture (later proved to be false)

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*This statement is based on what these systems do or achieve if considered as black boxes and not on how they are designed or how they function internally.*

*Fig. 1.8c, shows the next level (in terms of increasing complexity and utility) of processing, that of linear, dynamic processing (LDP). Here, the outputs depend on not only the present input values but also the past values of the inputs. It should be observed that LSP & LDP leads to linear models and the mapping from the input(s) to the output(s) represents a hyper-surface, a very restricted case of mapping. This kind of mapping has inherent limitations; from a systems point of view, a smaller bandwidth (to reduce the impact of unwanted signals or disturbances) leads to sluggish response in the time-domain and so on. For example, if we assume a plant represented by a linear first-order differential equation and controlled by a proportional control, the closed loop system model will be given by:

$$H_{Ls}(s) = \frac{k}{1+s\tau}$$

where the parameter $\tau$ affects both the bandwidth (inversely) and the rise time (proportionately). Thus, we are forced to make a compromise in choosing that parameter value.

Fig. 1.8d depicts the case of nonlinear, static processing (NLSP). Controllers and classifiers based on fuzzy expert systems (FES) and multi-layer feedforward neural networks (MLFFNN) using only present inputs fall under this category. The fact that the mapping resulting from NLSP can be more general than the mappings possible with LSP and LDP indicates that NLSP can provide better performance than LSP and LDP. However, NLSP leads to nonlinear dynamical systems (when used in a closed-loop system as in control) whose stability may not be provable mathematically. For example, in classical nonlinear control theory, we have the famous Aizerman's conjecture (later proved to be false)
regarding the stability of a closed-loop system consisting of a stable linear system in feedback with a memoryless nonlinearity.

Fig. 1.8e depicts the case of nonlinear, dynamic processing (NLDP). It should be observed that closed-loop systems consisting of systems (or plants) controlled by FESs and MLFFNNs as well as stand alone systems with FESs and or MLFFNNs along with feedback (or memory) fall under this category (see Fig. 1.9 for a general feedback architecture for the latter category. The readers may notice that this architecture bears resemblance to the general architecture for a synchronous machine, a well-known example of a nonlinear system with memory). The general mathematical characterization of such a system is in terms of a nonlinear differential equation connecting the input(s) and the output(s) written in the form of state equations. A simple example under this category is that of Hopfield network dynamics well known to the neural network community:

$$\frac{d\mu_i}{dt} = -\frac{\mu_i}{R_i} + \sum_{j}^{N} T_{ij} V_j[\mu_j] + I_i \quad ; \quad i = 1 \text{to} N \quad (1.13)$$

Here, $\mu_i$ is the state, $V_j[\bullet]$ are some nonlinear mappings of the state variables, $I_i$ is the bias-input, and $T_{ij}, R_i$ are constants.

Fig. 1.8f depicts the next (and perhaps the last in terms of complexity) level of processing, self-organizing systems (SOS). A specific example under this category is that of Adaptive Resonance Theory (ART) dynamics due to Grossberg et al. and given below:

$$\varepsilon \dot{x}_k = -x_k + (1-Ax_k)J_k^+ - (B+Cx_k)J_k^- \quad ; \quad k = 1 \text{to} M + N;$$

$$\dot{Z}_{ij} = k_1 f(x_j) \{-E_{ij}Z_{ij} + h[x_i]\} \quad i = 1 \text{to} M; \quad (1.14)$$

$$\dot{Z}_{ji} = k_2 f(x_j) \{-E_{ji}Z_{ji} + h[x_j]\} \quad j = M + 1 \text{to} M + N$$
where $f[\bullet]$ and $h[\bullet]$ represent nonlinear mappings of the independent variables. A complete description of the ART dynamics can be found in a chapter that deals exclusively with neural networks and the references. It is sufficient to note here that the state variables $x_j$ depend upon their past values and the weights $z_{ij}$ and $z_{ji}$ in a nonlinear fashion and the weights in turn depend on their past values and the state variables. However, the state changes much faster than the weights themselves; the weights $z_{ij}$ and $z_{ji}$ are slowly learnt from experience (I/O observations) and a higher level processing determines whether to learn a new pattern (produce a new class), whether to stop learning etc. Another example in self-learning systems is Model-Reference Adaptive Control, well known in the classical control area.

In this book, we are primarily concerned with nonlinear dynamical systems and self-learning or self-organizing systems and their applications. We show the different paradigms employed in categorizing the topics that fall under this broad category and the inter-relationship among the various topics in Fig. 1.10. Again, most textbooks deal with individual sub-topics (Linear, Time-invariant Digital Signal Processing, for example) in great detail. Here, we provide a concise but potentially useful overview of the various topics that can lead to
cross-fertilization of ideas from different topics. Also, the main premise of the book, that of passivity, to ensure the stability of nonlinear dynamical systems, is a property that is a part of the continuous-domain, where as, the preferred implementational vehicle for majority of applications is bound to be digital technology. Hence the emphasis on both the analog and the digital world.

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Fig. 1.10. Different paradigms used in linear & nonlinear dynamical systems

1.4 Advantages of Nonlinear Dynamics & Some Applications

Let us now look at why we should be concerned with nonlinear dynamics:

**Linear Dynamics can provide only a limited performance:** Linear filtering based on linear dynamics assumes that the desired information (or signal) occupies a separate frequency band from that of the disturbance or noise. If the noise and the signal share the same frequency spectrum, the noise cannot be minimized or eliminated without attenuating the desired signal as well. Also, as indicated earlier, there is an inherent trade-off between the performance in the time-domain and the performance in the frequency domain in linear dynamic processing. Nonlinear dynamic processing can overcome such limitations.

**Most systems are inherently nonlinear:** It should be observed that linearity is a condition imposed to make the problem mathematically tractable and easy to analyze. However, most real-world systems are nonlinear. For example, we can not hope to double the speed of an automobile from 60 MPH by simply pushing twice the amount of gas used at 60 MPH. Possibilities are the speed will only increase marginally, the engine will stall or break apart. Thus, it is better to deal with nonlinear effects as they occur.

**Higher level processing are based on nonlinear dynamics:** Intelligent information processing invariably involves nonlinear processing. In decision making, the input bandwidth (in terms of information and number of bits) is very large and the output bandwidth is very small (represented by one or very few bits). For example, around the end of 1990, an important question was if the country should go to war with Iraq to liberate Kuwait? Depending upon who was pondering the question, many considerations such as how many casualties we can tolerate, are we going to or can we afford to antagonize other countries that sympathize with Iraq, and so on would have formed the inputs to the decision making process and an one bit representing yes, we should, and no, we should not, would have been the output. This process is basically a nonlinear mapping from the inputs to the output. We could have made it dynamic by adding a feedback around the two possible outcomes as shown in Fig. 1.11. The addition of the feedback can make the whole decision process oscillatory, and in such a case, higher level processing that modifies the weights given to each argument, or adds more information has to be applied to make the output stable.
In summary, the reasons for going for nonlinear systems are to take into consideration real-world constraints and design high-performance systems. Therefore, it seems logical to carry information available from well-defined physical (real-world) elements into our design paradigm.

1.5 Organization of the Book

This book is organized into two parts. In Part I, we present fundamental concepts as related to nonlinear and learning systems. Part II deals with the application of nonlinear dynamics to areas such as signal processing, control, learning systems (neural and fuzzy systems and adaptive control), and chaotic systems.

Part I consists of eight chapters. In chapter 2, we provide a concise summary of the fundamentals of Signals and systems. We deal with both continuous and discrete linear time-invariant (LTI) systems. The basic definitions and the various mathematical tools are discussed. Issues such as infinite impulse response and delayed response are dealt from an engineering perspective. A broad overview of LTI signal processing and control is also provided. Finally, the similarities and the differences between these two application domains are listed.

In chapter 3, we consider two practical problems associated with nonlinear systems: 1) predicting the behavior of an analog system, through simulation on a digital computer, and 2) design of digital systems from analog system prototypes. We discuss the various transformations used for simulation and conversion of analog systems to digital systems, earlier application of these transformations in the design of LTI digital systems, the advantages and shortcomings of the transformations used, and the application of these transformations to nonlinear systems.

In chapter 4, we consider autonomous (or time-invariant) nonlinear systems from a classical, mathematical perspective. LTI system concepts are used to introduce the terminologies commonly used in nonlinear systems. We show that time-domain representation, especially the state space representation, is the most general and preferred form for representing such systems. Using simple, first order, nonlinear system models, we show that the nonlinear terms present in the model can lead to nonlinear systems with single or multiple equilibrium points. We will learn that due to the possibility for multiple equilibrium points, the term stability, if applied strictly, will be used to describe the equilibrium points and not the system. We will find that some of the equilibrium points will be stable and others are not. Therefore, we need to use modifiers such as global, local etc. while characterizing the stability of multiple equilibrium points of a system. We will also see that an equilibrium point may be stable but not absolutely stable. Or it may be absolutely stable, but the response may take a long time to reach the equilibrium point necessitating a restricted category of absolutely stable systems, called exponentially stable systems. We discuss the Lyapunov's direct method for checking the stability of an autonomous nonlinear system. We point out that it perhaps evolved by considering the energy left at any time in an otherwise undisturbed (by an external input) second-order physical system with initial stored energy and the procedure later mathematically extended to higher order systems. Further, we learn that the Lyapunov's direct method provides only a sufficiency condition in that the existence of a Lyapunov function indicates that the system is stable and the converse, our inability to come up with a suitable Lyapunov function, doesn't necessarily mean that the system is not stable.

For reasons that will become clear in later chapters, we consider the response of nonlinear systems to stored initial energy as different from the response to forced functions, especially DC excitation. This approach is different from the one taken by other researchers in this area. We learn that absolute stability defined on the basis of the response to stored initial energy doesn't necessarily imply stability under the presence of bounded forced excitation as is the case.

Figure 1-11. The process of intelligent decision making represented as a nonlinear mapping with feedback.
under LTI systems. Again, a discussion of the physical reasons for such different behavior has been deferred to later chapters.

In chapter 5, we use the concepts of power and energy, the ability to store (and return at a later time) or consume power to introduce the basic building blocks for electrical circuits. We first discuss LTI elements and later show how they can be easily and effortlessly applied to come up with meaningful nonlinear elements. Though the extension to nonlinear elements is rather straightforward, we will find that nonlinear elements can have well-defined, exotic I/O characteristics which can be used to explain easily the concepts such as equilibrium points, and the response that will result from circuits/dynamics based on such elements. We will find that those elements can be built in the analog domain or their I/O equations can simply be transformed to the digital domain for implementation using digital components.

In chapter 6, we consider the use of the various electrical-circuit-building-blocks introduced in the chapter 5 in forming complex electrical circuits and study the resulting dynamics. We first consider circuits made of LTI elements, the basic laws (Kirchhoff's current and voltage law and Tellegen's theorem) that apply when such elements are interconnected, and the restrictions that need to be observed while interconnecting such elements. We consider the I/O characteristics of such circuits and discuss concepts such as impedance and admittance functions and positive real functions. We will find that circuits with lossy elements lead to stable circuits. Since the lossy elements are also linear (which imply a power consumption ability that is proportional to the square of the current or voltage), we will find that such circuits also have the bounded-input, bounded-output property. We will consider building complex multiport circuits in a systematic manner and study their impedance and admittance matrices. We will find that a state-space representation of such a circuit carries more information (the I/O characteristics of the individual elements & the structural information) than is indicated by stability alone. We also discuss two important techniques in LTI network theory, that of impedance scaling and frequency scaling, that allows us to change the element values without affecting certain I/O characteristics of the network.

Of course, the reasons for looking at LTI electrical circuits are to learn how to carry some of that knowledge to the nonlinear electrical circuit domain and use them effectively. Thus, we also consider simple circuits made of nonlinear elements, the resulting dynamics and the behavior. Through these examples, we will find how issues such as multiple equilibrium can easily be explained, or how to form complex dynamics with desired characteristics. We will find the presence of lossy elements ensure stability of the resulting dynamics. In fact, we will find that the sum of the energy stored in the lossless elements at any given time is the true Lyapunov function for that nonlinear dynamics. Further, the sum of the power consumed by the lossy elements in the circuit corresponds to the negative of the derivative of the Lyapunov function and will always be non-negative. Thus, we will find that stability and passivity goes hand in hand as happens in LTI systems. We also look at the response to initial conditions and response to external excitation separately as they refer to two distinct physical scenarios. In the first case, we have the situation where we have finite stored energy to start with and wonder if that energy will be dissipated eventually. On the other hand, the second case implies a situation where power is being continually supplied by the external source and the question is how the circuit is handling that in-flow of power. Is it being dissipated as it comes along or gets stored? If so, how long this can continue? We will find that the flexibility in defining the I/O characteristics of the lossy (& other nonlinear) elements can also lead to the condition where the lossy elements are unprepared, so to speak, to absorb the power coming in leading to potentially bounded-input bounded-output (BIBO) instability. Thus, a circuit/dynamics that is absolutely stable may not necessarily be BIBO stable.

In chapter 7, we consider time-varying or non-autonomous systems and present some basic concepts of stability for time-varying systems (that are derived from a mathematical / analytical perspective). Again, the emphasis is to present the basic results in as simple terms as possible. The reasons for considering non-autonomous systems are that in a number of physical systems, the parameters associated with physical systems (that may or may not appear in the dynamical equations used to represent the systems) can vary with time. Some example parameters are temperature and pressure that play major role in many mechanical and aeronautical systems. Note in particular the two statements: 1) "parameters associated with physical systems," and 2) "vary with time," shown highlighted here. Usually, greater emphasis is placed on the latter statement in the analytical approach to dealing with non-autonomous systems with little or no attention to the first statement. Thus, time-varying is used to imply the appearance of terms such as \(t\), \(\exp[t]\), \(\sin[t]\) etc. in the dynamical equations with out attaching any physical significance to such terms and results based on such modeling techniques appear in the literature.

In chapter 8, we extend the building block concept to arrive at non-autonomous or time-varying (linear and nonlinear) elements and look at non-autonomous systems from a physical realization perspective. Again, this very simple approach leads to a number of powerful time-varying elements. These elements and circuits resulting from proper interconnection of such elements will be used to explain the various phenomena associated with non-autonomous systems and to arrive at a number of new results and techniques for designing stable non-autonomous systems.

In Part II, we deal with the application of nonlinear dynamics and the new paradigm to a number of application areas. Since the whole concept is new, only preliminary and important details are presented. However, the power of the new approach and its applicability to a wide range of areas will become obvious from the various chapters. We start with chapter 9 where we briefly look at different attempts and approaches to modeling of nonlinear systems. The methods
presented are derived from what one may call an abstract mathematical approach to modeling where any connection to physical systems is completely ignored. We first look at modeling memoryless nonlinear systems using orthonormal polynomials. From a study of this simple class of nonlinear systems, an important aspect of modeling nonlinear systems emerges: For a LTI system we have a single descriptor, such as the impulse response, to completely characterize the system. Therefore, we can use any suitable error criteria such as the $l$-2 norm and suitable weighting function (that emphasizes the errors in a particular region and de-emphasizes the errors in other regions) to approximate the response of the system or model the system. That is, we do not have to worry about the type of input that will be seen by the system. We will learn that this is not the case for nonlinear systems. For optimum modeling, the weighting function has to match the probability density function $P[x]$ of the input seen by the system.

Modeling nonlinear systems with memory is considered next. In particular, we look at two well-known techniques, Volterra series and Volterra-Wiener series representations and their limitations. Special models such as cascade models are briefly touched upon next. Finally, we look at neural networks' approach as yet another approach to modeling nonlinear systems. The original motivation behind neural networks is to use the knowledge from existing real-world biological systems (the brain). However, it is not clear how much of that knowledge has really been incorporated into artificial neural networks or for that matter, how much we really know about the brain to accomplish that goal. More on this topic is provided in a separate chapter.

In chapter 10, we use the new paradigm for the modeling and design of nonlinear systems in detail. Here we use only passive, nonlinear electrical elements\textsuperscript{10} as building blocks of complex, nonlinear passive electrical circuits and use the dynamics of such circuits (expressed in terms of the I/O characteristics of the various elements) as templates for systems to be designed. We first discuss the general philosophy and precedence for the new philosophy from other areas. Next we derive the general form of complex, first-order and second-order stable nonlinear dynamics derived using the new approach followed by the general form for higher order nonlinear dynamics. We prove the global, asymptotic stability properties of nonlinear dynamics derived from passive circuits. We provide a network theoretic interpretation of the difference between absolute stability and BIBO stability and show how that information can be effectively used to design nonlinear dynamical systems with both global absolute stability and BIBO stability. Finally, we look at some well-known NLDIEs and their circuit equivalents.

In chapter 11 we consider nonlinear dynamical control design using the building block & the reverse engineering approach. Nonlinear control is a very complex topic and it is not possible to cover all aspects of nonlinear control in one or two chapters. Thus, in this chapter, we try to discuss only the important issues and provide a distinctly different perspective on nonlinear control using the new building block approach. We start with a summary of important aspects of modeling. Then we consider the tasks of asymptotic stabilization, set-point control and tracking control. Through some simple examples we demonstrate what is and what is not possible in nonlinear control.

Issues involved in modeling, design, and control of nonlinear (autonomous and non-autonomous) systems are more complex than what we encounter in LTI systems. Frequency domain techniques are not applicable for most (if not all) of the nonlinear systems and we have to work in the time-domain with careful consideration given to input amplitude etc. since the principle of superposition doesn’t apply here. The state-space model is an ideal candidate for representing nonlinear systems. However, the fact that nonlinear operators or primitives have to be present in any nonlinear model coupled with the availability of huge number of nonlinear primitives and a variety of interconnections (or architectures) make the model selection extremely difficult.

To overcome the problems associated with the selection of the nonlinear primitives and the architecture, most research in nonlinear systems use state-space models which are simple extensions of the LTI model (with apparently no consideration to their connection to real-world plants). For example we have the single-input single-output (SISO) canonical model,

$$
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t) \\
\vdots \\
\dot{x}_n(t)
\end{bmatrix}
= 
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
\vdots \\
x_n(t)
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
\vdots \\
g[x]
\end{bmatrix} u(t)
$$

(1.15)

where the nonlinearities are confined to the last (or one) state equation and ‘a multiplicative nonlinearity’ is assumed for the input’s role. Similarly, the strict feedback model:
response and not an arbitrary entity. It is true that we still have tremendous proper interconnection of well-defined components that leads to a desired very logical, as a plant or a system or a process (that needs to be controlled) is a systems using the fundamental concepts of power and energy and looking at modeling basic elements or building blocks needed for complex nonlinear systems. These models correspond to what is known as direct form structures (and their variations) that are notorious for their high sensitivity (of the response to variations) that are notorious for their high sensitivity (of the response to scaled inputs and so on since linearity and superposition does not apply to nonlinear systems. Further work is needed to handle issues such as these examples, we can see that if a plant is not stable, we can go only so far if the number and / or location of the inputs are constrained. Similarly, a model that represents an electrical circuit (a physical system) can only be controlled (whether it is set-point or tracking control) only to certain extent that depends on the internal architecture of the circuit including how the inputs are given to the plant. Thus, it is clear that we can not take a real-world plant with restrictions on inputs and achieve what ever we want from control alone. Further, we can not always generate the input (to the plant) needed for perfect tracking through specifying the output alone. We need to know either all the state variables or a certain specific state variable (that depends on the plant) and use it and the plant model to estimate other state variables and hence the tracking input.

We discuss adaptive control in chapter 12. First, we provide a primer to adaptive control. We next show the connection between dynamics of an adaptive control with parameters that needs to be adapted and the rules for convergence of those parameters using the circuit dynamics perspective. Again, adaptive control is a well-researched topic with a number of separate books devoted to it and the aim of this chapter is to show how the circuits approach can be used to explain the important concepts in adaptive control and make the design of such systems much easier.

We consider the use nonlinear circuits for signal processing in chapter 13. We explain the concept using some specific examples and show how the circuits approach can be used to design useful signal processing systems using knowledge about the signals, the task at hand, and architectures well-known in LTI signal processing. Once again, this is just a beginning in the use of the new approach for signal processing. Further work is needed to handle issues such as scaled inputs and so on since linearity and superposition does not apply to nonlinear systems. In chapter 14, we consider signal estimation and detection using nonlinear circuits. Again, we use specific examples to show the power of the new approach. First, we consider Kalman filtering for signal estimation and show
that the present approach for Kalman filter design leads to circuits that are time-varying and circuits that can become unstable. Further, we update the Kalman gain using a priori information that does not include the error between the observed signal and the estimated signal. That is, the Kalman algorithm leads to architectures that are not completely coupled leading to the possibility of instability. Using the circuits and the reverse engineering approach, we show how new estimators (and architectures) can be designed that are highly coupled and guaranteed to be stable. In fact, such an approach leads to time-varying and nonlinear architectures.

We also consider the problem of estimation using the concept of attractors and show how proper architectures can be arrived in a simple and straightforward manner using the building block. Again, we explain the whole concept using an example, that of estimating a bipolar binary signal corrupted by Gaussian white noise. A circuit is designed to have those two possible values, ±1, as the attractors. Thus, in this example, we use both passive as well as active elements to arrive at a suitable architecture. Design of such circuits will be discussed further in the context of neural networks in the next chapter.

In chapter 15, we provide a new perspective into neural networks, both feedforward and feedback networks. In section 15.2, we provide a small primer on neural networks. In addition to providing necessary details (that will be of use to those who are not familiar with neural networks) in a concise manner, we try to remove the mystery surrounding these networks and try to look at them in a more critical and objective way. Thus, we concentrate on what they can really do, how they do it, what are the advantages and disadvantages, etc. One key aspect of neural networks we try to highlight in this and the next section is that they represent special classes of nonlinear circuits, and hence we can learn quite a bit by understanding nonlinear circuits and their dynamics. In section 15.3, we consider recurrent neural networks (RNNs), or neural networks with internal feedback, and discuss some of the well-known models. In the case of recurrent neural networks, stability, or the lack of it, is a major concern, and we discuss some of the existing approaches to prove the stability of RNNs. This section highlights the slow progress in the design of RNNs due to the lack of structured methods to obtain RNN architectures with guaranteed stability property and the problems in training such networks. In section 15.4, we discuss a new approach based on the building block concept (seen in earlier chapters for designing complex, stable nonlinear circuits) to obtain new and complex RNN architectures and develop training or learning algorithms. We also show that existing RNN architectures can be derived by placing specific constraints on the architectures obtained using the building block approach. We also indicate that in RNN architectures, no energy function is being minimized, but a situation of power balance (between sources and sinks) is reached.

In chapter 16, we consider another emerging, hot area in intelligent systems - that of fuzzy logic (FL), fuzzy expert systems (FESs) and fuzzy controllers (FCs), mainly from the perspective of nonlinear systems. We first provide a simple and concise introduction this area. We provide only the salient details about functional and implementational aspects of fuzzy expert systems and fuzzy controllers and omit mathematical details as much as possible. This section will be of interest to those who want to learn about these areas from a practical perspective with out having to spend an enormous amount of time wading through theorems, lemmas, and proofs. The readers will find that fuzzy logic casts in an analytical model the approaches taken by human beings in dealing with day-to-day problems that are nonlinear by nature and the resulting solutions. What will emerge from the discussions in this section is that fuzzy logic and fuzzy expert systems are in fact excellent vehicles for specifying multi-input / multi-output static mappings (MIMO). The beauty of fuzzy logic lies in the relatively simple and easy approach with which MIMO nonlinear functional mappings of practical relevance can be described and implemented.

The use of fuzzy logic in control applications is considered in section 16.2. We give some examples that illustrate how fuzzy logic can be used to design control laws and discuss the performance of systems controlled by fuzzy controllers. From the examples we will find that fuzzy controllers (based on some appropriate knowledge) can indeed do a good job. Perhaps the important point to learn is that since the controller is in general nonlinear, better performance can be obtained from different fuzzy controllers based on the knowledge from different experts. That is, the nonlinearity by itself makes it possible to have more than one near-optimal control law and leads to control laws that are robust. However, the nonlinear nature of the control law makes the closed-loop system nonlinear even if the open-loop system is LTI and it is very difficult to prove the stability of such nonlinear systems. We present some results from classical control theory to show the efforts that went into proving the stability of such systems and the difficulties encountered. Furthermore, as indicated earlier, fuzzy logic leads to static (memoryless) control laws which can lead to abrupt changes in the control law that may not be acceptable in practice. Also, when the number of inputs is large, the design complexity of fuzzy controllers also increases. Then, there is the question of adaptability or learning in fuzzy expert systems.

In sections 16.3 and 16.4, we provide some solutions to these problems. In section 16.3, we learn how fuzzy logic can be combined with neural nets to retain the best of both worlds: use of experts’ knowledge when available and adaptability based on observed data. We also discuss a neural network architecture known as Cerebellar Model Articulation Controllers (CMAC) Neural Network which was proposed independently some twenty five years ago and its similarity to fuzzy expert systems. In section 16.4, we show how the building block concept can be used to overcome the stability problem, introduce memory into otherwise static fuzzy expert systems and provide the ability to deal with number of input variables. The network approach thus leads to next generation of fuzzy controllers called feedback or recurrent fuzzy controllers and can find use in a number of applications.
In chapter 17, we consider certain classes of nonlinear circuits and their response to initial conditions and / or external forcing functions. Through a number of circuit examples and the behaviors of such circuits, we introduce two important areas of research in nonlinear dynamical systems, that of chaotic systems and fractal systems. The emphasis here is on how to interpret such systems from nonlinear circuits' perspective. Also, such an interpretation will help us in building complex chaotic systems and design controllers for otherwise chaotic systems.

We first consider circuits made of nonlinear passive elements only driven by external sinusoidal sources and discuss the various possible responses. We next consider nonlinear circuits with no independent sources but consisting of nonpassive elements. We discuss two special cases, nonpassive elements with continuously differentiable characteristics and nonpassive elements represented by piece-wise linear models. The generation of limit cycles and chaotic signals from such circuit architectures are discussed. Next we consider first-order discrete domain nonlinear dynamics with just one parameter and discuss how such systems can lead to chaos. We also consider continuous domain nonlinear circuits that lead to such discrete domain dynamics and provide a different perspective on the behavior of such systems. Then, we introduce fractal systems and discuss their connection to nonlinear electrical circuits and continuous domain dynamics. Finally, we show how the building block approach can be used to design synchronous chaotic systems for transmission and retrieval of sensitive information. Once again, this design example illustrates how complex systems can be easily designed using the building block approach with little or no effort.

The discussions about chaotic systems, and fractal systems given here are in no way complete nor follow the conventional wisdom about such systems. The purpose here is simply to introduce the readers the ideas in a manner related to the nonlinear networks' concepts, the main thrust of this book.

1.6 Notes and References

The importance of LTI filter structures has been well documented in a number of books. For example, [L.T. Bruton, 1980] for Active RC filters and [A. Antoniou, 1979] for Digital filters. Some of the material in section 1.2 are based on the excellent paper by [M. Vidyasagar, 1986]. Though this paper is ten years old, it describes well the direction of nonlinear control theory. Additional information on Losslessness and passivity as applied to nonlinear circuits can be found in [J.L. Wyatt, Jr., et. al., 1981, and 1982]. Information on bond graphs and their application to dynamic systems can be found in [D.C. Karnopp, et. al., 1990].