ELECTRONICS I
January 23, 1995

I. The figure below shows a voltage source connected to the input of an amplifier. \( R_s \) is the source resistance. \( R_i \) and \( C_i \) are the input resistance and input capacitance of the amplifier, respectively. Derive an expression for \( V_i(s)/V_s(s) \) and show that it is a low-pass STC type. Find the 3-dB frequency for the case where \( R_s = 5k\Omega \), \( R_i = 200k\Omega \), and \( C_i = 20\mu F \). (25 pts).

\[
\frac{V_i}{V_s} = \frac{Z_i}{R_s + Z_i}
\]

\[
Z_i = \frac{1}{\frac{1}{R_i} + \frac{1}{\jmath \omega C_i}} = \frac{1 + \jmath \omega R_i C_i}{R_i}
\]

\[
\frac{V_i}{V_s} = \frac{R_i}{1 + \jmath \omega R_i C_i} \left[\frac{R_s + \frac{R_i}{1 + \jmath \omega R_i C_i}}{R_i}\right]
\]

\[
\omega_B = \frac{R_s + R_i}{R_i R_s C_i} = \frac{1}{\omega_0 R_i R_s C_i}
\]

\[
\omega_B = \frac{205 \text{ k}\Omega}{20,000 \text{ k}\Omega} = \frac{205}{20,000} \frac{1}{10^3} \text{ rad/s} = 1.63 \times 10^6 \text{ rad/s}
\]

\[
\omega_B = \frac{205}{20,000} \frac{1}{2\pi} = 1.63 \times 10^6 \text{ Hz}
\]

\[
\omega_B = 1.63 \times 10^6 \text{ Hz}
\]
II. In the diagram below the op-amp is assumed to be ideal and all resistors are assumed to be equal to 1Ω for simplicity. Find an expression of $v_o$ in terms of $v_1$ and $v_2$. Hint: Use the principle of superposition.

\[ A = \infty \]

\[ n_0^+ = \frac{n_2}{2} \quad n_0^- = \frac{n_2}{2} \]

\[ i_1 = \frac{n_2 - n_1}{2} = n_1 - \frac{n_2}{2} \]

\[ n_0^+ = n_2 - n_1 - R \left( i_A + i_1 \right) = n_2 - n_1 - \left( n_1 - \frac{n_2}{2} + n_1 - \frac{n_2}{2} \right) \]

\[ n_0^- = n_2 - n_1 - \left( 2n_1 - \frac{3}{2}n_2 \right) = \frac{5}{2}n_2 - 3n_1 \]

\[ n_2 = 0 \rightarrow n_0^+ = n_0^- = 0 \rightarrow i_1 = n_1 \rightarrow i_A = -n_1 \rightarrow n_0^+ = -n_1 \rightarrow n_0^- = -n_1^2 - 2n_1^2 = -3n_1^2 \]

\[ n_1 = 0 \rightarrow n_0^+ = n_0^- = \frac{n_2}{2} \rightarrow i_1 = -\frac{n_2}{2} \rightarrow n_0^+ = n_2 \rightarrow i_A = -\frac{n_2}{2} \rightarrow n_0^- = n_0^+ - R(i_1 + i_A) = \frac{n_2}{2} - \left( \frac{3n_2}{2} \right) = \frac{5}{2}n_2 \]

\[ n_0 = n_0^+ + n_0^- = \frac{5}{2}n_2 - 3n_1 \]
IV. Consider a voltage follower (buffer amplifier) built with an ideal op-amp except for having a finite voltage gain $A$. Calculate the value of the close-loop gain for $A=10, 100,$ and $1000$. In each case calculate the % error in gain magnitude compared to the gain obtained with an ideal op-amp. (15 pt)

\[ N_o = A \left( N_\text{in} - N_\text{in} \right) \]

\[ N_\text{in} = N_o \]

\[ \frac{N_o}{N_\text{in}} = \frac{N_\text{in} - N_o}{N_\text{in}} \]

\[ N_\text{in} \left[ 1 + \frac{1}{A} \right] = N_o \]

\[ G = \frac{N_o}{N_\text{in}} = \frac{1}{1 + \frac{1}{A}} \]

<table>
<thead>
<tr>
<th>$A$</th>
<th>$G ,(A=\infty)$</th>
<th>$G$</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>0.909</td>
<td>9.1%</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
<td>0.990</td>
<td>1%</td>
</tr>
<tr>
<td>1000</td>
<td>1</td>
<td>0.999</td>
<td>0.1%</td>
</tr>
</tbody>
</table>

\[ \% \text{ error} = \left| \frac{G - G \,(A=\infty)}{G \,(A=\infty)} \right| = \left| \frac{1 + \frac{1}{A}}{-1} \right| = \frac{1}{A} \frac{A}{A+1} = \frac{1}{1+A} \]
III. A designer has available voltage amplifiers having an input resistance of 10kΩ, an output resistance of 1kΩ, and an open-circuit voltage gain of 10. The signal source has a 10kΩ resistance and provides 10 mV rms signal. If it is required to provide a signal of at least 2V rms to a 1kΩ load, how many amplifier stages are required? For this number of amplifiers, what is the actual output voltage? (30pts)

\[
\begin{align*}
V_L &= \frac{1}{2} A \frac{N_{i3}^2}{2} \frac{N_{i2}^2}{2} \\
V_L &= \frac{A^2}{2} \left(\frac{10}{11}\right)^2 \frac{V_S}{2} \\
V_L &= \frac{1}{2} A \left(\frac{10}{11}\right)^{n-1} \frac{V_S}{2} \quad \text{where } n = \# \text{ of stages}
\end{align*}
\]

We want:

\[
\frac{V_L}{V_S} = 200 = \frac{1}{2} \left(\frac{10}{11}\right)^{n-1} = \frac{1}{2} \frac{10}{11}^{n-1} = \frac{11}{20}^{n-1}
\]

\[
\frac{V_L}{V_S} = 200 = \frac{11}{20} \left(\frac{100}{11}\right)^m \Rightarrow \text{solve for minimum } n \text{ needed}
\]

\[
\left(\frac{100}{11}\right)^m = \frac{1000}{11} = 363.6
\]

\[
\Rightarrow \text{for } n = 3, \left(\frac{100}{11}\right)^3 = 755
\]

\[
\text{for } n = 2, \left(\frac{100}{11}\right)^2 = 83
\]

\[
\Rightarrow 3 \text{ stages needed}
\]
I. (25 pts) A voltage amplifier has the transfer function

\[ A_v = \frac{100}{(1 + \frac{jf}{10^1})(1 + \frac{jf}{10^7})} \]  \hspace{1cm} (1)

Using the Bode plots for low-pass and high-pass filters, sketch the Bode plot for \(|A_v|\). Give approximate values for the gain amplitude at \(f = 10, 10^2, 10^3, 10^4, 10^5, 10^6, \) and \(10^7\) Hz. Find the bandwidth of the amplifier (defined as the frequency range over which the gain remains within 3 dB of the maximum value).

Bandwidth \(B = \omega L \omega^2 = 9900\,\text{Hz}\)
Calculate $v_0$ in terms of $v_1, v_2, v_3$ for the circuit below. Assume ideal op-amps.

\[ v_x = -v_1 - 2v_2 \]

\[ v_0 = -(-v_1 - 2v_2) - 3v_3 \]

\[ v_0 = v_1 + 2v_2 - 3v_3 \]
IV. (25 pts) For the circuit shown below, derive an expression for the input resistance of the inverting amplifier taking into account the finite open-loop gain of the op-amp.

\[ R_{\text{in}} = \frac{V_+}{i_+} \]

\[ V_+ = A(V_+ - V-) \]
\[ V_+ = 0 \]
\[ \Rightarrow V_+ = -A V_- \]
\[ = V_+ - i_+ R_2 \]
\[ \Rightarrow i_+ R_2 = (A+1) V_- \]
\[ \Rightarrow V_- = i_+ R_2 / (1 + A) \]

Now, \[ V_+ = i_+ R_1 + V_- \]
\[ = i_+ R_1 + \frac{i_+ R_2}{A+1} \]
\[ \Rightarrow R_{\text{in}} = \frac{V_+}{i_+} = R_1 + \frac{R_2}{A+1} \]
IV. (25 pts) For the circuit below, calculate the transfer function \( T(s) = \frac{v_0(s)}{v_i(s)} \). Is it of the low-pass or high-pass type? Give an expression for the 3 dB frequency.

\[
\begin{align*}
\frac{v_0}{v_i} &= \frac{R_2}{R_1 + j\omega L} = \frac{-R_2/R_1}{1 + j\omega L/R_1} = \frac{-R_2/R_1}{1 + \frac{\omega}{\omega_{3dB}}} \\
\Rightarrow K &= -\frac{R_2}{R_1} \\
\omega_{3dB} &= \frac{R_1}{L} \\
\text{Low-pass filter} &+5 \text{ points}
\end{align*}
\]
III. (25 pts) For the circuit shown below, find \( v_o \) in terms of \( v_1 \) and \( v_2 \) assuming an ideal op-amp. For

\[ v_1 = 10\sin(2\pi X 60t) - 0.1\sin(2\pi X 10000t), \text{volts} \]  \hspace{1cm} (2)

and

\[ v_2 = 10\sin(2\pi X 60t) + 0.1\sin(2\pi X 10000t), \text{volts} \]  \hspace{1cm} (3)

Find \( v_o \).

Find the input resistance seen by the differential input signal \( v_2 - v_1 \).
II. (25 pts) The circuit shown below (left) can be used to implement a transresistance amplifier. Find the value of the input resistance $R_i$, the transresistance $R_m$, and output resistance of the transresistance amplifier $R_o$. If the signal source shown on the right below is connected to the input of the transresistance amplifier, find its output voltage. Does it depend on the load resistance? Why?

\[ R_i = \frac{V_i}{i_i} = 0 \]

\[ R_o = 0 \]

\[ i_i = 0.5 \, mA \]

\[ V_o = -10 \, k\Omega \times 0.5 \, mA = -5 \, V \]

$V_0$ does not depend on $R_L$ because $R_o = 0$. (2:3)
Problem 1: The circuit shown below utilizes a 10 kiloohms potentiometer to realize an adjustable-gain amplifier. Derive an expression for the gain as a function of the potentiometer setting x. Assume the op amp to be ideal. What is the range of gains obtained? Show how to add a fixed resistor so that the gain range can vary from 1 V/V to 11 V/V. What should the resistor value be?

\[ V_o = V_i \left[ 1 + \frac{(1-x) \times 10}{x \times 10} \right] \]
\[ = V_i \left( 1 + \frac{1}{10} \right) \]
\[ V_o = V_i / x \]

The gain obtained ranges from +1 to +\( \infty \).

For gain range of +1 to +11, need to add a resistor R to ground end of pot so that when x = 0,
\[ \text{Gain} = 1 + \frac{10 \times 10}{R} = 11 \Rightarrow R = 1 \text{k}\Omega \]
ELECTRONICS I
June 5, 1996

I. In the circuit below, express $v_0$ in terms of $v_1$ and $v_2$. Assume the op-amps are ideal. Calculate $v_0$ if

$$v_1 = 3\sin(2\pi \times 60t) + 0.01\sin(2\pi \times 1000t)\text{Volts},$$

(1)

and

$$v_2 = 3\sin(2\pi \times 60t) - 0.01\sin(2\pi \times 1000t)\text{Volts}.$$  

(2)

Show your work.

\[
\begin{align*}
N_2 &= 0, \quad N_{01} = -10N_3 = 10N_1, \\
N_1 &= 0 = N_3, \quad N_{02} = -10N_2 \\
N_6 &= N_{0\,1} + N_{0\,2} = 10(N_3 + N_2) = 10(N_3 - N_0) \\
N_1 - N_2 &= +0.02\sin(2\pi \times 1000t) \\
\Rightarrow N_0 &= +0.2V \sin(2\pi \times 1000t)
\end{align*}
\]
III. (25 pts) In the circuit below, the op-amp is ideal except that it has a finite A. Express \( v_o \) in terms of \( v_x, v_y \), and the open-loop gain A. If \( v_y \) is grounded, what is the input resistance seen by \( v_x \). Express your answer in terms of R and A.

\[
\begin{align*}
V^+ &= \frac{V_y}{2} \\
V^- &= \frac{V_y}{A} \\
V_o &= A \left( V^+ - V^- \right)
\end{align*}
\]

\[
\begin{align*}
V^+ &= \frac{V_y}{2} \\
V^- &= \frac{V_o}{A} \\
V_o &= \frac{V^- - V^+}{A}
\end{align*}
\]

\[
I = \frac{V_x - V_y}{R} = \frac{V^- - V^+}{R}
\]

\[
\Rightarrow V_o = 2V^- - V_x = 2 \left( \frac{V^- - V^+}{A} \right) - V_x
\]

\[
V_o = \frac{V_y - V_x}{1 + \frac{2}{A}}
\]

\[
F_{\text{in}} = R \omega \gg A
\]

\[
R_{\text{in}} = R \left( \frac{A + 2}{A+1} \right)
\]
II. (25 pts) In the circuit shown below, assume that all op-amps are ideal. Give an expression for \( v_0 \) in terms of \( v_1, v_2, \) and \( v_3 \). Show all intermediate steps for full credit.

\[
\begin{align*}
N_x & = \frac{-3}{2} N_3 + \theta \\
N_y & = -N_1 - 2N_2 - 2N_X + \theta \\
N_\theta & = \frac{-N_1 - 2N_2 + 3N_3}{2} + \theta \\
N_0 & = -N_\theta = N_1 + 2N_2 - 3N_3 + \theta
\end{align*}
\]
IV: (25 pts) A measurement of the open-loop gain of an internally compensated op-amp at very low frequencies show it to be $4.2 \times 10^4$ V/V. At 100 kHz, it is 76V/V. Estimate the values for $A_o$, $f_b$, and $f_t$. Remember that your answers must be in Hz for $f_b$ and $f_t$.

\[
A_o = 4.2 \times 10^4 \text{ V/V} \quad \text{(1)}
\]

\[
A = \frac{A_o}{1 + j \frac{f}{f_b}}
\]

\[
|A| = \frac{A_o}{\sqrt{1 + \left(\frac{f}{f_b}\right)^2}}
\]

\[
f_b = \frac{4.2 \times 10^4}{\sqrt{1 + (\log_{10} f_b)^2}} \Rightarrow f_b = 0.181 \text{ kHz}
\]

\[f_t = A_o f_b = 4.2 \times 10^4 \times 0.181 \text{ kHz} \quad \text{(5)}
\]

\[f_t = \frac{7.69 \text{ kHz}}{10}
\]
(25 pts) A voltage amplifier with an input resistance of 10 kΩ, when driven with a current source of 1 μA and a source resistance of 100 kΩ, has a short-circuit output current of 10 mA and an open-circuit output voltage of 10 V. When driving a 4 kΩ load, what are the voltage gain, current gain, and power gains expressed in dB?

\[ V_o = 10 \times \frac{4}{4+1} = 8 \text{ V} \]

\[ A_v = \frac{V_o}{V_i} = \frac{8+3}{\text{V}_{\text{in}}^2} = 8.88 \text{ V/V or } 58.9 \text{ dB} \]

\[ A_i = \frac{I_o}{I_i} = \frac{V_o/R_L}{V_o/\text{R}_{\text{in}}} = \frac{4}{10} = 2200 \text{ A/A} \]

\[ \theta_p = \frac{V_o^2}{R_L} = 14.36 \text{ V}^2 / \text{W} = 62.9 \text{ dB = } \frac{1}{2} \left( A_v + A_i \right) \]