Hwk #6
ECE 622
Due Monday Feb. 27, 2012

① The Pauli matrices X, Y satisfy the relation

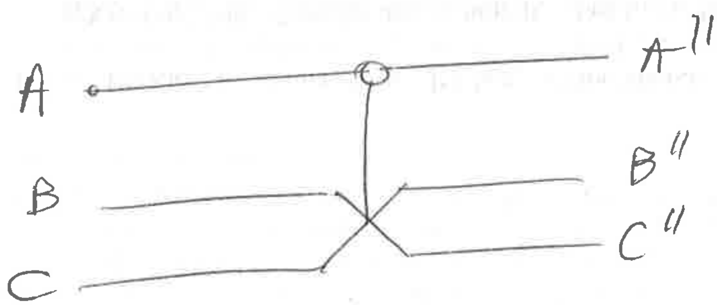
$$XYX = -Y$$

Show it. Using this result, show that

$$X R_y(\theta) X = R_y(-\theta)$$

where $R_y(\theta)$ is the rotation matrix by an angle θ around the y axis.

② A Fredkin gate is defined as follows



when $A=0$ $\quad B''=B$, $C''=C$

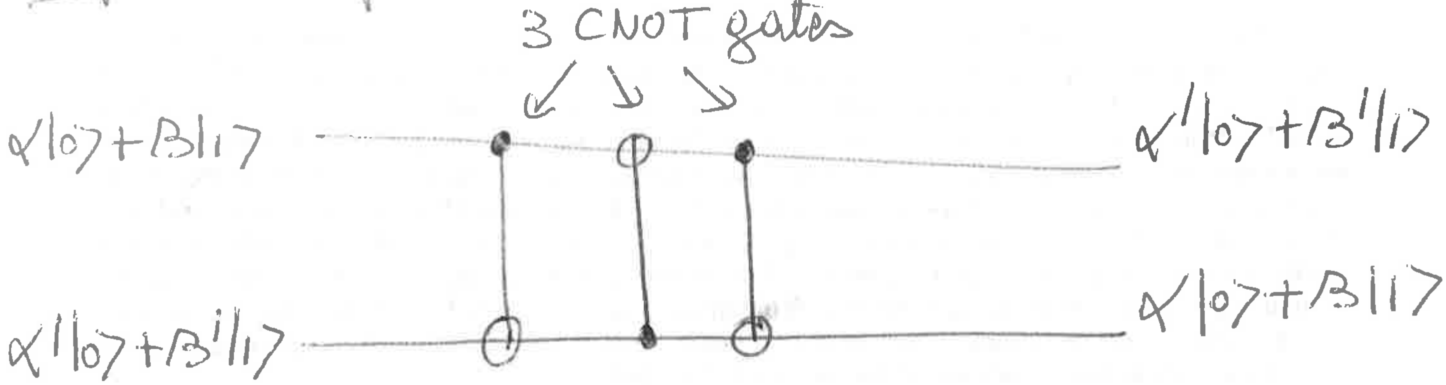
when $A=1$ $\quad B''=C$, $C''=B$ } swapping

If A, B, C are classical bits, give the truth table for the Fredkin gate. Show that the Fredkin gate is reversible.

Show that $B'' = \bar{A}B + AC = \bar{A}B \oplus AC$

Show that $C'' = \bar{A}C + AB = \bar{A}C \oplus AB$

(3) Proof that the following circuit will swap 2 qubits



(4) Consider the following two Bell states representing a system of 2 qubits in C_4.

$$|\beta_{11}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

(a) What is the explicit form of the two bras (1×4 row vectors) associated to these (4×1) kets?

(b) Show explicitly that $|\beta_{01}\rangle$ is orthogonal to $|\beta_{11}\rangle$

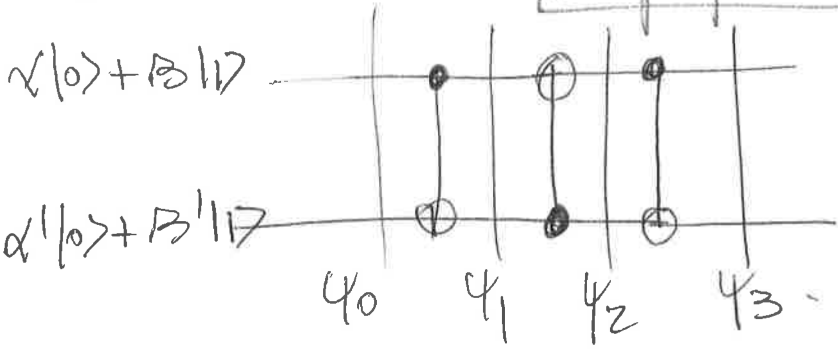
(c) What is the average value of the tensor product

$$\sigma_y \otimes \sigma_z$$

in the 2 qubit-state

$$\left[\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right] \otimes \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right] ?$$

Swap operation



$$\psi_0 = (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha'|0\rangle + \beta'|1\rangle)$$

$$\psi_0 = \alpha|0\rangle \otimes (\alpha'|0\rangle + \beta'|1\rangle) + \beta|1\rangle \otimes (\alpha'|0\rangle + \beta'|1\rangle)$$

$$\psi_1 = \alpha|0\rangle \otimes (\alpha'|0\rangle + \beta'|1\rangle) + \beta|1\rangle \otimes (\alpha'|1\rangle + \beta'|0\rangle)$$

$$\psi_1 = \alpha\alpha'|0\rangle \otimes |0\rangle + \alpha\beta'|0\rangle \otimes |1\rangle + \beta\alpha'|1\rangle \otimes |1\rangle + \beta\beta'|1\rangle \otimes |0\rangle$$

$$\psi_2 = \alpha\alpha'|0\rangle \otimes |0\rangle + \alpha\beta'|1\rangle \otimes |1\rangle + \beta\alpha'|0\rangle \otimes |1\rangle + \beta\beta'|1\rangle \otimes |0\rangle$$

$$\psi_3 = \alpha\alpha'|0\rangle \otimes |0\rangle + \alpha\beta'|1\rangle \otimes |0\rangle + \beta\alpha'|0\rangle \otimes |1\rangle + \beta\beta'|1\rangle \otimes |1\rangle$$

$$(\alpha'|0\rangle + \beta'|1\rangle) \otimes \alpha|0\rangle + (\alpha'|0\rangle + \beta'|1\rangle) \otimes \beta|1\rangle$$

$$\boxed{\psi_3 = (\alpha'|0\rangle + \beta'|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle)}$$

Swap operation