(I) Prove that

$$\sigma_x \sigma_y \sigma_z = iI,$$

where $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the three Pauli matrices and $I$ is the identity matrix.
(II) If a spin is in the qubit state (which is properly normalized)

\[ |\psi> = \frac{1}{\sqrt{10}} (3|0> - |1>), \]

what is the probability to find a value of \( \frac{\hbar}{2} \) if you measure the y-component of the spin.
(III) Show that the matrix \( \sqrt{\text{NOT}} \) given by

\[
\begin{pmatrix}
\frac{1+i}{2} & \frac{1-i}{2} \\
\frac{i}{2} & \frac{-1+i}{2}
\end{pmatrix}
\]  

is unitary.
(IV) What is the 4X1 column vector resulting from the following tensor product

\[(H \otimes \sigma_x)(|0 > \otimes |1 >)\]  \hspace{1cm} (4)

where H is the Hadamard matrix

\[H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\] \hspace{1cm} (5)
(V) Compute the following Kronecker product $H \otimes H \otimes H$ where $H$ is the Hadamard Matrix.
(VI) What is the average value of the tensor product

\[ H \otimes \sigma_z \]  

(6)

(where \( H \) is the Hadamard matrix and \( \sigma_z \) is the Pauli matrix) in the 2-qubit state given by

\[
\left[ \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \otimes \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \right].
\]  

(7)