Feedback Effect on Amplifier with 2 poles

\[ A(\omega) = \frac{A_0}{(1 + \frac{\omega}{\omega_{pH1}})(1 + \frac{\omega}{\omega_{pH2}})} \]

\[ A_f(\omega) = \frac{A(\omega)}{1 + B_f A(\omega)} \]

\[ A_f(\omega) = \frac{A_0}{(1 + \frac{\omega}{\omega_{pH1}})(1 + \frac{\omega}{\omega_{pH2}})} \]

\[ A_f(\omega) = \frac{A_0}{[1 + \frac{\omega}{\omega_{pH1}}][1 + \frac{\omega}{\omega_{pH2}}] + B_f A_0} \]

\[ A_f(\omega) = \frac{A_0}{1 + B_f A_0 + \Delta \left[ \frac{1}{\omega_{pH1}} + \frac{1}{\omega_{pH2}} \right] + \Delta^2 \left( \frac{1}{\omega_{pH1} \omega_{pH2}} \right)} \]

\[ A_f(\omega) = \frac{A_0}{1 + B_f A_0} \left[ 1 + \frac{\Delta}{1 + B_f A_0} \left[ \frac{1}{\omega_{pH1}} + \frac{1}{\omega_{pH2}} \right] + \frac{\Delta^2}{\omega_{pH1} \omega_{pH2} (1 + B_f A_0)} \right] \]
\[ A_f(\gamma) = \frac{A f_0}{1 + \frac{\Delta^2}{\omega_o Q} + \frac{\Delta^2}{\omega_o^2}} \]

Define
\[ \omega_o = \omega_{PH1} \omega_{PH2} \left( 1 + \beta f A_0 \right) \]

or
\[ \omega_o = \left[ \omega_{PH1} \omega_{PH2} \left( 1 + \beta f A_0 \right) \right]^{-1} \]

\[ Q \text{ is defined such that} \]
\[ \frac{1}{\omega_o Q} = \left[ 1 + \frac{1}{\omega_{PH1} + \omega_{PH2}} \right] \]

\[ Q = \frac{\left( 1 + \beta f A_0 \right) \omega_{PH1} + \omega_{PH2}}{\omega_o \omega_{PH1} + \omega_{PH2}} \]

\[ Q = \frac{\omega_o}{\omega_{PH1} + \omega_{PH2}} \]

\[ \rightarrow \]
\[ A_f(\gamma) = \frac{A f_0}{1 + \frac{\Delta^2}{\omega_o Q} + \frac{\Delta^2}{\omega_o^2}} \]

Pole mixing

Proximity of the other pole
location of new poles of amplifier with feedback

\[ 1 + \frac{1}{w_0^2} + \frac{1}{w^2} = 0 \quad \text{or} \quad D^2 + \frac{\omega_0}{\Delta + \omega_0} = 0. \]

\[ D = -\frac{\omega_0}{\Delta} \pm \frac{1}{2} \sqrt{\left(\frac{\omega_0}{\Delta}\right)^2 - 4 \omega_0^2} \]

Pole mixing

\[ D = -\frac{\omega_0}{\Delta} + \left(\frac{\omega_0}{\Delta}\right) \sqrt{1 - 4Q^2} \]

\[ Q = \frac{\omega_{PH1} \omega_{PH2} (1 + Bf A_0)}{w_{PH1} + w_{PH2}} \]

With no feedback, \( Q \) reaches a minimum

\[ Q_{\text{min}} = \frac{\sqrt{w_{PH1} w_{PH2}}}{w_{PH1} + w_{PH2}} \]

\[ D = -\frac{\omega_{PH1}}{\Delta}, -\omega_{PH2} \quad \text{(original poles)} \]

If \( Bf > 0 \), \( Bf A_0 > 0 \), \( Q > 1 \)

until \( 1 - 4Q^2 = 0 \) \( \Rightarrow Q = 0.5 \)

\[ D = -\frac{\omega_0}{\Delta} \pm \frac{1}{2} \frac{\omega_{PH1} + \omega_{PH2}}{\ell} \]
Second-Order Filter Functions

**Generic Form**

\[
T(s) = \frac{Q_2 s^2 + Q_1 s + Q_0}{s^2 + \frac{Q_0}{Q} + \omega_0^2}
\]

**Magnitude of \( T(s) \) for Low-Pass and First-Order Filter when**

\[
Q_1 = 0 \\
Q_2 = 0 \\
Q_0 \neq 0
\]

**Alien**

\[
T(s) = \frac{Q_0}{s^2 + \frac{Q_0}{Q} + \omega_0^2}
\]

\( s = j\omega \)

\[
T(j\omega) = \frac{Q_0}{-\omega + \omega_0 + \frac{\omega_0^2}{Q} + j\omega}
\]

\[
|T(j\omega)| = \frac{Q_0}{\sqrt{(-\omega + \omega_0 + \frac{\omega_0^2}{Q})^2 + (\omega_0 + \frac{\omega_0^2}{Q})^2}}
\]
Let's calculate magnitude of $\|T\|_{2\to 1}$ at $\omega = j \omega$

$$|T(j \omega)| = \frac{a_0}{\sqrt{(a_0^2 - \omega^2)^2 + \left( \frac{a_0 \omega}{\varphi} \right)^2}}$$

Boole-Plot $|T(j \omega)|$ versus $\omega$

If denominator reaches a minimum, $|T(j \omega)|$ will reach a maximum.

Let us calculate the frequency at which this occurs. We rewrite

$$|T(j \omega)| = \frac{a_0}{\sqrt{g(\omega)}}$$

$g(\omega)$ reaches a minimum when $\frac{d}{d \omega} g(\omega) = 0$

i.e.,

$$2(a_0^2 - \omega^2)(-2\omega) + 2\left( \frac{a_0 \omega}{\varphi} \right)^2 \left( \frac{a_0}{\varphi} \right)^2 = 0$$

Rearranging we get

$$-2(a_0^2 - \omega^2) = a_0^2 / \varphi^2$$

$$\omega = a_0 \sqrt{1 - \frac{1}{2 \varphi^2}}$$

$\Rightarrow \omega = \omega_{\text{max}}$
Bode Plot (Magnitude)

\[ |T(j \omega_{\text{max}})| = T_{\text{max}} \]

\[ \omega_{\text{max}} = \omega_0 \sqrt{1 - \frac{1}{2Q^2}} \]

Since \( Q > 0 \), \( \omega_{\text{max}} < \omega_0 \)

\[ T_{\text{max}} = \frac{a_0}{\sqrt{(\omega_0^2 - \omega_{\text{max}}^2)^2 + \left(\frac{a_0 \omega_{\text{max}}}{Q}\right)^2}} \]

i.e.,

\[ T_{\text{max}} = \frac{a_0}{\sqrt{\left[\omega_0^2 - \omega_0^2 \left(1 - \frac{1}{2Q^2}\right)\right]^2 + \left(\frac{a_0}{Q}\right)^2 \omega_0^2 \left(1 - \frac{1}{2Q^2}\right)}} \]
$$T_{\text{max}} = \sqrt{\frac{\omega_0^4}{w_0^2 \left[ 1 - \frac{1}{2Q^2} \right]^2 + \frac{\omega_0^4}{Q^2 \left( 1 - \frac{1}{2Q^2} \right)^2}}}$$

$$T_{\text{max}} = \frac{\omega_0}{w_0^2 \sqrt{\frac{1}{Q^4} + \frac{1}{Q^2} - \frac{1}{2Q^2}}}$$

$$T_{\text{max}} = \frac{\omega_0}{w_0^2 \sqrt{\frac{1}{Q^2} - \frac{1}{4Q^4}}}$$

$$T_{\text{max}} = \frac{\omega_0 \, Q}{w_0^2 \sqrt{1 - \frac{1}{4Q^2}}}$$

$$T_{\text{max}} = T(\Delta \omega = 0) \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}}$$

$$T_{\text{max}} < T(0) \text{ as long as } \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} < 1$$
So, there is no bump in $|T(j\omega)|$ until

$$\frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} = 1$$

i.e.,

$$Q^2 = 1 - \frac{1}{4Q^2}$$

$$\rightarrow Q^2 + \frac{1}{4Q^2} = 1$$

This occurs when $Q = \frac{1}{\sqrt{2}} = 0.707$

This value of $Q$ corresponds to what is called the maximally flat transfer function

For $Q > \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = 0.707$

There will be a bump (boss) appearing in the Bode plot of the magnitude of $|T(j\omega)|$