Concept of Quantum bits (qubits)

Contrary to classical bits (Cherts) which are either 0 or 1, qubits are a coherent superposition of both 0 and 1.

\[ qubit = \alpha |0\rangle + \beta |1\rangle \]

where \(|0\rangle\) denotes the state in which the qubit has value 0 and \(|1\rangle\) "1" 1.

\(\alpha, \beta\) are complex numbers whose square magnitudes denote the probability that if a measurement is performed on the qubit, it will be found to have a value 0 and 1, respectively.

The qubit must ultimately succumb to the fate of having a definite classical value (0 or 1) when it is measured. (WAVEFUNCTION COLLAPSE)

We must have

\[ |\alpha|^2 + |\beta|^2 = 1 \]

Prior to the measurement, the qubit does not have a definite value; it is both 0 and 1, at all times. It is therefore called a superposition state.
\( \alpha, \beta \) are continuous variables whose magnitudes can take any value between 0 and 1 and whose phases can also take any value between 0 and 2\( \pi \).

On the other hand, the states \( |0> \) and \( |1> \) correspond to digital binary bits. Thus, quantum computation is neither quite analog computing, nor digital, but something in between.

For two-level quantum systems used as qubits, the book will use the convention of identifying the state \( |0> \) with the vector \((1, 0)\), and similarly \( |1> \) with \((0, 1)\).

These vectors in \( \mathbb{C}^2 \) are also written as column vectors:

\[
|0> \leftrightarrow \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
|1> \leftrightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

Hence, a qubit \( \alpha |0> + \beta |1> = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \)

We need to review the properties of vector spaces of vectors of this form. This is done in Chapter 2, Section 2.1, on linear algebra.
\[ \psi_n = \sqrt{\frac{2}{W}} \sin \left( \frac{m_n \pi x}{W} \right) \]
\[ E_n = \frac{m^2}{2m^*} \frac{\hbar^2}{W} \frac{n^2}{W^2} \]

\[ \int_0^W |\psi_n|^2 = 1 \]

\[ E_2 = 4E_1 \]
\[ E_3 = 9E_1 \]
Polarized Photons

Fig. 8.2 A linearly polarized photon consists of an oscillating electric field and an oscillating magnetic field that are perpendicular to each other and to the direction of propagation.

Fig. 8.3 How a birefringent crystal can be used to separate photons based on their polarization.

Electro-optic modulator

\[
\begin{pmatrix}
1 \\
1
\end{pmatrix}
= \begin{pmatrix}
1 \\
0
\end{pmatrix}
+ \begin{pmatrix}
0 \\
1
\end{pmatrix}
\quad \text{(45° pol.)} \quad \text{(z pol.)} \quad \text{(χ pol.)}
\]

\[P_0 \propto \left| \begin{pmatrix}
e^{ik_1 L} \\
e^{ik_2 L}
\end{pmatrix} \right|^2 = 4 \cos^2 \frac{(k_1 - k_2)L}{2}.
\]

Birefringence of material is controlled by \( V \).

This effect has also been observed with birefringent liquids (polar molecules).

The liquid becomes birefringent when placed in an electric field. This is referred to as the Kerr effect.
Spin and the Exclusion Principle

Fig. 12.1 Stern-Gerlach apparatus for the measurement of the vertical component of the spin.

\[ \text{[1s}^2 2s^2 2p^6]\] filled shell.
**ZEEMAN EFFECT**

\[ \vec{B} \neq 0 \]

\[ \vec{B} = 0 \]

\[ \Delta E = 2 \mu_B B \]

\[ s = -1/2 \]

\[ s = +1/2 \]

\[ H = -\vec{\mu}_B \cdot \vec{B} = \frac{g^*}{2} \mu_B B \]

\[ \Delta E = g^* \mu_B B \]

\[ \mu_B = \text{BOHR MAGNETON} = \frac{e \hbar}{2m_0} \]

\[ g^* = \text{Landé Factor} = 2 \text{ in vacuum} \]

In InAs, for B = 1 Tesla: \[ \Delta E \sim 0.1 \text{ meV} \]

\[ g^* = -14 \]

4 meV = 1 THz

0.1 meV = 25 GHz

T = 300 K = 26 meV

0.1 meV = 1 K

Search for materials with large \( g^* \)

DMS (Dilute Magnetic Materials: Ga Mn As, ....)
LIST OF REFERENCES


- http://www.idquantique.com/. This website describes the release of hardware to implement quantum cryptography protocols:

- A user friendly software to write and test quantum algorithms is described at http://tph.tuwien.ac.at/ oemer/doc/qr-rplog/index.html

- http://www.cse.iitk.ac.in/news/primality.html

Prof. Manindra Agarwal and two of his students, Nitin Saxena and Neeraj Kayal (both BTech from CSE/IITK who have just joined as Ph.D. students), have discovered a polynomial time deterministic algorithm to test if an input number is prime or not. Lots of people over (literally!) centuries have been looking for a polynomial time test for primality, and this result is a major breakthrough, likened by some to the P-time solution to Linear Programming announced in the 70s.