

Hint for problem 3.

It can be shown (Book by Nielsen and Chuang, p. 174) that any unitary operator U can be written as

$$U = e^{i\alpha} R_z(\beta) R_y(\theta) R_z(\gamma)$$

where R_z, R_y are the rotation matrices defined in class

$$R_y(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} e^{-i\gamma/2} & 0 \\ 0 & e^{i\gamma/2} \end{bmatrix}$$

H is unitary, your problem is to find $\alpha, \beta, \theta, \gamma$ in the expression above so that

$$H = U(\alpha, \beta, \theta, \gamma)$$