I. (30 pts): The voltage gain of an amplifier is described by the following transfer function:

\[ A(s) = \frac{10^8 s^2}{(s + 10)(s + 10^2)(s + 10^6)} \]

- Write \( A(s) \) in the generic form \( A_M F_L(s) F_H(s) \) and give the expressions for \( A_M \), \( F_L(s) \), and \( F_H(s) \).
- What is the midgap gain \( A_M \) of the amplifier (in dB)? What are the zeroes and poles of \( A(s) \) on the low and high frequency sides?
- Make a Bode plot of the magnitude of the transfer function \( A(s) \). Use the attached log-log plot. Make sure to indicate clearly the locations of all the zeroes and poles of the amplifier on the Bode plot.
- From your Bode plot, find the approximate values of the gain (in dB) at the angular frequencies \( \omega = 1, 10, 10^2, 10^3, 10^4, \text{ and } 10^5 \text{ rad/s} \).
II. (30 pts): For the amplifier shown below

- What type of configuration is the amplifier (circle correct answer): (a) common emitter, (b) common base, (c) or common collector?  
- Draw the small ac equivalent circuit of the network (hybrid - π model) and derive an analytical expression for the midband voltage gain of the amplifier. Neglect the effects of the $r_x$ and $r_0$ when drawing the small AC equivalent circuit.
- Use the Gray-Searles procedure to derive an analytical expression of the low frequency pole associated to the coupling capacitor $C_1$. 

\[ \frac{V_o}{V_s} = \frac{V_{o^*}}{V_{s^*}} \cdot \frac{N_i^*}{N_i} \cdot \frac{N_c^*}{N_c} \] 

\[ R_V = R_{CH} + (\beta + 1) R_E \]
III. (20 pts): Find the analytical expressions for the $h_{21}$ and $g_{12}$ parameters of the following feedback network. Make sure to specify the units of each parameter.

\[ h_{21} = \frac{I_2}{I_1}, \quad V_2 = 0 \]

\[ I_2 = -\frac{R_2}{R_2 + R_3} I_1 \]

\[ h_{21} = -\frac{R_2}{R_2 + R_3} \]

\[ g_{12} = \frac{I_1}{I_2}, \quad V_1 = 0 \]

\[ I_1 = \frac{R_4}{R_4 + R_3 + (R_1/R_2)} I_2 \]

\[ I_1 = -\frac{R_2}{R_1 + R_2} I_1 \]

\[ I_1 = -\frac{R_2 R_4}{(R_1 + R_2) \left[ R_3 + R_4 + (R_1/R_2) \right]} \]

\[ g_{12} = \frac{I_1}{I_2}, \quad V_1 = 0 \]

\[ g_{12} = \frac{R_2 R_4}{(R_1 + R_2) \left[ R_3 + R_4 + (R_1/R_2) \right]} \]
IV. (20 pts): A shunt-series feedback amplifier uses a basic amplifier with a gain of 500A/A and an input and output resistances $R_i$ and $R_o$ equal to 0.5 kΩ and 100 kΩ, respectively. The feedback factor $\beta_f$ is 0.2A/A. Find the gain $A_f$, the input resistance, $R_{i,f}$ and output resistance $R_{o,f}$ of the amplifier with feedback.

$$A_f = \frac{A}{1 + B_f A} = \frac{500}{1 + 0.2(500)} = \frac{500}{101}$$

$$A_f = 5$$

$$R_{i,f} = R_i / (1 + B_f A) = \frac{0.5 \, \text{k}\Omega}{101} = 5\, \Omega$$

$$R_{o,f} = R_o (1 + B_f A) = 100 \, \text{k}\Omega \times 101 = 10,100 \, \Omega$$
V. (30 pts): Derive the transfer function $T(s) = V_o/V_i$ for the filter below. Assume the op-amp is ideal.

(1) Explain qualitatively why this filter is a high-pass filter.

(2) Show that $T(s)$ is of the form $\frac{a_3 s^2}{s^2 + \frac{a_2}{\omega_0} s + \omega_0^2}$ and determine the expressions of $a_2, \omega_0,$ and $Q$ as a function of the components of the circuit (resistors and capacitors).
\begin{align*}
\frac{N_0}{N_i} &= \left(\frac{1 + \delta R_1 C_3 + \delta C_4}{R_1}\right) \left(\frac{1 + \delta R_2 C_3}{\delta R_2 C_3}\right) - \frac{1}{R_1} \left(1 + \delta R_1 C_3\right) \\
&= N_i \sqrt{\frac{\alpha}{\alpha'}}
\end{align*}

\begin{align*}
\frac{N_0}{N_i} &= \frac{\delta^2 R_1 R_2 C_3 C_4}{(1 + \delta R_1 (C_3 + C_4))(1 + \delta R_2 C_3) - \delta R_2 C_3 (1 + \delta R_1 C_3)} \\
&= \frac{\delta^2 R_1 R_2 C_3 C_4}{1 + \delta^2 R_1 R_2 C_3 (C_3 + C_4) + \delta^2 R_1 R_2 C_3 (C_3 + C_4) - \delta R_1 (C_3 + C_4) - \delta^2 R_2 C_3 - \delta^2 R_1 R_2 C_3}
\end{align*}

\begin{align*}
T(\delta) &= \frac{\delta^2 R_1 R_2 C_3 C_4}{1 + \delta^2 R_1 R_2 C_3 C_4 + \delta R_1 (C_3 + C_4)} \\
&= \frac{\delta^2}{\delta^2 + \frac{\alpha}{\alpha'} + \frac{\omega_0^2}{\omega^2} + \omega_0^2}
\end{align*}

\begin{align*}
\alpha^2 &= \frac{\omega_0^2}{\omega^2} + \frac{\omega_0^2}{\omega^2} + \frac{\omega_0^2}{\omega^2} \\
&= \frac{1}{\sqrt{K_1 K_2 C_3 C_4}} \\
&= \frac{1}{\sqrt{K_1 K_2 C_3 C_4}} \\
\frac{\omega_0}{\omega} &= \frac{1}{\sqrt{K_2 (C_3 + C_4)}} \\
\Rightarrow \quad Q &= \frac{R_2 (C_3 + C_4)}{K_2 (C_3 + C_4)}
\end{align*}
VI. (30 pts): Consider the circuit below used to build an oscillator. The op-amp is assumed to be ideal, i.e., it has an infinite impedance resistance looking into the positive and negative terminals of the op-amp. Relate first \( v_x \) to \( v_o \) using the fact that for the ideal op-amp \( v_+ = v_- \). Then, set \( v_x = v_o \). You will end up with an equation of the form
\[ v_o (a + jb) = 0. \] (2)

- The only way that equation can be satisfied for a non-zero \( v_o \) is if both \( a \) and \( b \) are equal to zero. The condition \( b = 0 \) will give you the frequency of oscillation of the circuit in terms of the passive elements in the network. Write the explicit analytical expression for the frequency of oscillation.
- The condition \( a = 0 \) will give you the relation between the passive elements in the network for the oscillations to be observed. Write down the explicit relation which must be satisfied by the passive components for the circuit to oscillate.

\[ I_0 = \sigma L_2 I_1 \]
\[ I_1 = I_2 = \frac{N_0}{\sigma L_2} \]
\[ N_0 = -RI_2 = -N_0 \frac{R}{\sigma L_2} \]
\[ I_3 = \frac{0 - N_0}{\sigma L_1} = \frac{N_0 R}{\sigma^2 L_1 L_2} \]
\[ I_y = I_1 + I_2 \]
\[ = \frac{V_0}{\delta L_2} + \frac{V_0 \cdot R}{\delta^2 L_1 L_2} \]
\[ = \frac{V_0}{\delta L_2} \left( 1 + \frac{R}{\delta L_1} \right) \]

\[ V_X = N_a - \frac{1}{\delta C} I_y \]

\[ N = N_X - \frac{V_0 R}{\delta L_2} - \frac{1}{\delta C} \frac{V_0}{\delta L_2} \left( 1 + \frac{R}{\delta L_1} \right) \]

\[ N_0 \left[ 1 + \frac{R}{\delta L_2} + \frac{1}{\delta^2 L_2 C} \left( 1 + \frac{R}{\delta L_1} \right) \right] = 0 \]

\[ 1 - \frac{R}{\omega L_2} + \frac{1}{\omega^2 L_2 C} + \frac{R}{\delta^3 L_1 L_2 C} = 0 \]

\[ \left( 1 - \frac{1}{\omega^2 L_2 C} \right) - \frac{R}{\omega L_2} + \frac{R}{\omega^3 L_1 L_2 C} = 0 \]

\[ \omega = \frac{1}{\sqrt{L_2 C}} \]

\[ - \frac{R}{\omega L_2} + \frac{R}{\omega^3 L_1 L_2 C} = 0 \]

\[ \frac{R}{\omega L_2} = \frac{R}{\omega^3 L_1 L_2 C} \]

\[ \frac{1}{L_2} = \frac{L_{EC}}{L_1 L_2 C} \]

\[ \therefore L_1 = L_2 \]
VII. (20 pts): For each of the inverter transfer characteristics below, calculate the noise margins for both the low (NML) and high state (NMH) of the inverter. Indicate how you calculate these values from each graph. The low state is defined as low voltage range for the input and the high state is defined as the high voltage range of the output.
VIII. (20 pts): In the circuit shown below, assume that the forward bias across the EB junction of Q1 and Q3 is 0.7 V when they start conducting and answer the following ten questions.

1. When $v_i = 0.2$ V, the transistor Q1 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.

2. When $v_i = 0.2$ V, $v_0$ will be equal to $0$ V.

3. When $v_i = 0.2$ V, the transistor Q3 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.

4. When $v_i = 5$ V, the transistor Q1 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.

5. When $v_i = 5$ V, the transistor Q3 will be in what mode of operation (circle the correct answer), (a) forward active, (b) reverse active, (c) cutoff, or (d) saturation mode.

6. When $v_i = 2.5$ V, the pn Base-Emitter junction of Q1 is forward biased, TRUE or FALSE.

7. When $v_i = 2.5$ V, Q1 and Q3 are both in the cutoff mode, TRUE or FALSE.

8. When $v_i = 5$ V, $V_{B1,C1} = V_{B3,E3} = 0.8$ V and $V_{C3,E3} = 0.1$ V, TRUE or FALSE.

9. When $v_i = 0.2$ V, the current $I_R$ mostly flows into the emitter of Q1, TRUE or FALSE.

10. When $v_i = 0.4$ V, $V_{B1,C1} = V_{B3,E3} = ...$ V.