1. (20 pts) In the circuit below, a measurement shows that the emitter voltage is 1.2V. Find $V_B$, $V_C$, $I_E$, $I_B$, $I_C$, and the value of $\beta$ and $\alpha$. Neglect the effect of the Early voltage ($\lambda=0$).

**Diagram:**

- $V_E = +5V$
- $R_E = 5\,\text{k}\Omega$
- $R_B = 50\,\text{k}\Omega$
- $R_C = 5\,\text{k}\Omega$

**Calculations:**

$V_{EB} = V_E - V_B = 1.2V$

$I_E = \frac{5 - 1.2}{5} = 0.76\,\text{mA}$

$V_B = 1.2 - 0.7 = 0.5\,\text{V}$

$I_B = \frac{0.5 - 0}{50\,\text{k}\Omega} = 0.01\,\text{mA}$

$I_C = I_E - I_B = 0.75\,\text{mA}$

$V_C = -5 + 0.75 \times 5 = -1.25\,\text{V}$

$\beta = \frac{I_C}{I_B} = \frac{0.75}{0.01} = 75$

$\alpha = \frac{\beta}{\beta + 1} = \frac{75}{76} = 0.987$
III. (30pts) For the circuit below, find the DC collector current and the DC voltage at the collector. Assume $\beta$ is 100 and that $V_{BE}$ is 0.7 V. Use the T-model and draw the small signal equivalent circuit of the amplifier. Neglect the effect of $r_0$. Determine the voltage gain $V_0/V_I$.

**DC**

$$I_C = 0.99 \text{ mA}$$

$$V_C = R_E I_E + V_{BE} + R_B I_B$$

$$= (1 \times 0.175) + 0.7 + (0.01 \times 100) = 1.875 \text{ V}$$

**AC**

$$i_c = \frac{V_i}{Z_c} = \frac{V_i}{0.2k\Omega}$$

$$\frac{V_o - V_i}{10k\Omega} + V_i + \frac{V_o}{0.1k\Omega} = 0$$

$$0.01V_o - 0.01V_i + 0.99V_i + 0.1V_i = 0$$

$$\Rightarrow \frac{V_o}{V_i} = -44.9 \text{ V/V}$$
2. (20 pts) Measurements on the circuit below produce the labeled voltages as indicated. Find the value of β for the transistor.

\[ I_E = \frac{10 - 7}{1k} = 3 \text{ mA} \]

\[ I_E = I_C + I_B = 3 \text{ mA} \]

\[ V_C = 3 \text{ mA} (1k) = 3 \text{ V} \]

\[ I_B = \frac{6.3 - 3}{100k} = 33 \mu A \]

\[ \frac{I_E}{I_B} = \beta + 1 = \frac{3 \text{ mA}}{33 \mu A} = 90.9 \]

\[ \beta = 89.9 \]
3. (30 pts) The BJT as an amplifier

- (a) In the circuit below, assume $V_{BE} = 0.7V$ and calculate the value of the collector current and the voltage at the collector.
- (b) Using the T-model of the BJT and neglecting the effect of $r_o$, draw the small signal equivalent circuit of the amplifier.
- (c) Using the results of part (b), calculate the voltage gain of the amplifier, $v_o/v_i$.

\[ I_C = 0.99 \times 0.5mA = 0.495mA \]
\[ V_C = I_C R_C + R_B I_B \\
\]
\[ = 0.99 \times 0.175 + 0.9 \\
\]
\[ = 2.26V \]

\[ I_C = \frac{V_i}{(R_C + R_E)} = \frac{V_i}{300} \]

\[ V_C = \frac{V_i}{I_E} = 50.5V \]

\[ N_o - N_i^2 + k i_e + \frac{N_o}{20k} = 0 \]

\[ V_o = -90V/V \]
4. (30 pts) The BJT as an amplifier

- (a) Is the amplifier below a (1) common-emitter, (2) common-base, or (c) common-collector configuration? (circle the correct answer)
- (b) Including the effects of \( r_0 \) and using the T-model of the transistor, draw the small-signal equivalent circuit of the amplifier. Derive the analytical expression for the small-signal voltage gain \( A_v = \frac{v_o}{v_i} \) of the amplifier.
- (c) Derive the expression for the input resistance \( R_{in} \).
8. (30 pts) For the NMOS amplifier below, replace the transistor with its T equivalent circuit while neglecting the effect of $r_0$. Calculate the voltage gains $v_s/v_i$ and $v_d/v_i$.

\[ \frac{v_s}{v_i} = \frac{R_s}{R_s + \frac{1}{g_m}} \left[ 1 + g_m R_s \right] \]

\[ v_d = -g_m v_{gs} R_D \]

\[ v_{gs} = \frac{1/g_m v_i}{1 + R_s} = \frac{v_i}{1 + g_m R_s} \]

\[ \therefore \frac{v_d}{v_i} = \frac{-g_m R_D}{1 + g_m R_s} \]
V. For the circuit shown below, both transistors are characterized by the following parameters $\mu_n C_{ox} = 20 \, \mu A/V^2$, $V_i = 1V$, $\lambda = 0$, $L = 10 \, \mu m$, and $W = 30 \, \mu m$. Find the labeled current and voltage.

\[ V_2 = 3V \]

\[ I_2 \]

\[ Q_1, Q_2 \text{ identical and operating in saturation with equal drain currents} \]

\[ \Rightarrow V_{GS1} = V_{GS2} \]

\[ \Rightarrow V_2 - V_2 = 3 - V_2 \Rightarrow V_2 = \frac{1.5}{2}V \]

\[ I_2 = \frac{1}{2} \times 20 \times \frac{30}{10} (1.5 - 1)^2 = 7.5 \, mA \]
7. (30 pts) For the amplifier circuit shown below:

- Which type of amplifier configuration is it? **common source, common gate, or common drain.** Circle your answer.
- Use the hybrid-π equivalent AC circuit for the transistor and draw the small signal equivalent circuit for the amplifier.
- Give an analytical expression for the input resistance \( R_{in} \), as shown on the diagram.
- Give an analytical expression for the overall voltage gain \( \frac{v_0}{v_{sig}} \).
6. (20 pts) The transistor in the circuit below has a $V_t = 1V$, $k_n \frac{W}{L} = 0.4mA/V^2$ and $\lambda = 0$.

Find the labeled node voltage $V_1$.

\[ \frac{V_1 + 5}{100} = I = \frac{1}{2} \times 0.4 \left( 5 - \frac{\infty}{V_1} \right)^2 \]

\[ I = 0.04 \times 5 \div 0.036 \text{ mA} \]

Because $V_{GS} < V_T \rightarrow \text{cut-off}$

Only acceptable solution

\[ V_1 = -5 + 100I = -1.4V \]

\[ I = \frac{1}{2} \frac{k^1 W}{L} \left( V_{GS} - V_T \right)^2 \]

\[ = \frac{1}{2} \frac{k^1 W}{L} \left( 0 - V_T - V_T \right)^2 \]

\[ = \frac{1}{2} \frac{k^1 W}{L} \left( -(-5 + 100I) - 1 \right)^2 \]

\[ I = 0.2(4 - 100I)^2 \rightarrow I_1, I_2 \text{ above} \]
5. (20 pts) The NMOS transistors in the circuit below have $V_i = 1V$, $k_n \frac{W}{L} = 2mA/V^2$ and $\lambda = 0$.

Find the labeled node voltages, $V_1$, $V_2$, and $V_3$.

\[ 10 - V_3 - V_1 = I \]  
\[ I_{D1} = \frac{V_1}{1} = \frac{1}{2} \cdot 2 \left( V_3 - V_2 - 1 \right)^2 \]  
\[ \rightarrow V_0 = (V_3 - V_2 - 1)^2 \]  
\[ I_{D2} = \frac{V_1}{1} = \frac{1}{2} \cdot 2 \left( V_2 - V_1 - 1 \right)^2 \]  
\[ \rightarrow V_3 - V_2 - 1 = V_2 - V_1 - 1 \rightarrow V_1 = 2V_2 - V_3 \]  
\[ 2V_2 - V_3 = 10 - V_3 \rightarrow V_2 = 5V \]  
\[ \rightarrow V_1 = (4 - V_1)^2 \rightarrow V_1^2 - 8V_1 + 16 = 0 \]  
\[ V_1 = 6.85 \text{ or } 2.45 \]  
\[ V_3 = 10 - 2.45mA \cdot 1k \Omega = 7.55V \]
VI. The MOSFET in the circuit below has \( V_i = 1\, \text{V}, \) \( K = 0.4\, \text{mA/V}^2, \) and \( V_A = 40\, \text{V}. \) (a) Assuming \( R_D = 50\, \text{k}\Omega, \) find \( R_S \) and \( R_C \) so that \( I_D = 0.1\, \text{mA}. \) (b) Find the values of \( g_m \) and \( r_0 \) at the bias point. (c) If terminal \( Y \) is grounded, find the voltage gain from \( X \) to \( Z \) with \( Z \) open-circuited. (d) If terminal \( X \) is grounded and terminal \( Z \) is connected to a current source delivering a current of 10\( \mu\text{A} \) and having a resistance of 100 \( \text{k}\Omega, \) find the voltage signal at \( Y. \) Neglect \( r_0 \) in that case.

\[
\begin{align*}
V_D &= 0 \\
V_Z &= \frac{5-0}{0.1} = 50 \text{k}\Omega \\
V_{GS} &= 1.5 \text{V} + Z \\
R_S &= \frac{-1.5-(-5)}{0.1} = 35 \text{k}\Omega \\
R_6 \text{ undetermined but must be high} \\
\text{Choose } R_6 &= 40 \text{ M}\Omega \\
V_i &= 0 \\
R_D &= \frac{5-0}{0.1} = 50 \text{k}\Omega \\
R_S &= \frac{-1.5-(-5)}{0.1} = 35 \text{k}\Omega \\
A_{in} &= \frac{(R_s//\eta_0)}{(R_s//\eta_0) + \frac{1}{q} m} = 0.928 \text{V/V} + 2 \\
l_0 &= \frac{V_A}{I_D} = 400 \text{k}\Omega + 2 \\
id &= i_i \frac{(R//R_s)}{(R//R_s) + \frac{1}{q} m} + 2 \\
V_D &= 1.5 \text{V} + 2 \text{V} \\
V_{GS} &= 1.5 \text{V} + 2 \text{V} \\
U_6 &= R_D \text{id} = 9.12 \times 50 = 456 \text{mV} + 2 \text{V}
\end{align*}
\]
8. (25 pts) The amplifier circuit shown below is biased in the saturation mode of operation with a value of the transconductance $g_m = 1$ mA/V. Assume $\lambda = 0$.

- Use the hybrid-\pi model of the transistor and draw the small signal equivalent circuit for the amplifier.
- Calculate the voltage gain $\frac{v_o}{v_{sig}}$.
6. (25 pts) In the circuit below, the NMOS has a $|V_t|$ of 0.9 V and a $V_A$ of 50 V and operates with $V_D = 2V$.

(a) Using the hybrid-$\pi$ model of the MOS transistor, draw the small signal equivalent circuit of the amplifier. Include the effect of $r_0$.

(b) What do the DC value $V_D$ becomes if the current $I$ is changed to 1 mA? Neglect the effects of $V_A$ in the expression of $I_D$.

\[ I_D = \begin{cases} \frac{k_i}{2} \frac{w}{L} (V_{G S 1} - V_t)^2 & \text{For } I_D = 500 \mu A \\ \frac{k_i}{2} \frac{w}{L} (V_{G S 2} - V_t)^2 & \text{For } I_D = 1 \text{ mA} \\ \end{cases} \]

\[ \frac{I_D}{I_{D1}} = \frac{(V_{G S 1} - V_t)^2}{(V_{G S 2} - V_t)^2} \Rightarrow V_{G S 2} = V_t + \sqrt{2(V_{G S 1} - V_t)} \]

\[ V_{G S 2} = 0.9 + \sqrt{2(2-0.9)} = 2.5V \]

$V_D = V_{G S 1}$

$V_D = V_{G S 2}$
5. (25 pts) The NMOS transistors in the circuit below have $V_t = 1\text{V}$, $\mu_nC_{ox} = 120\mu\text{A}/\text{V}^2$, $\lambda = 0$, and $L_1, L_2, L_3 = 1\ \mu\text{m}$.

Find the required values of the gate widths $W_1$ of $Q_1$, $W_2$ of $Q_2$, $W_3$ of $Q_3$ to obtain the voltage and current values indicated on the figure.

\begin{equation}
I_D = \frac{1}{2} k_m \frac{W}{L} (V_{GS} - V_t)^2
\end{equation}

\begin{equation}
k_m = \frac{1}{L} \mu_n C_{ox}
\end{equation}

$V_{GS_1} = 1.5\ \text{V} \rightarrow 120\ \mu\text{A} = \frac{1}{2} \cdot 120 \cdot \frac{W_1}{L_1} (1.5 - 1)^2 \rightarrow W_1 = 8\ \mu\text{m}$

$V_{GS_2} = 3.5 - 1.5 = 2 \rightarrow 120 = \frac{1}{2} \cdot 120 \cdot \frac{W_2}{L_2} (2 - 1)^2 \rightarrow W_2 = 2\ \mu\text{m}$

$V_{GS_3} = 1.5\ \text{V} \rightarrow W_3 = W_1 = 8\ \mu\text{m}$
5. (20 pts) In the circuit below, the enhancement-NMOS transistor has a threshold voltage $V_t = 1V$. Furthermore $k_i (W/L) = 0.4 \text{ mA/V}^2$ and $\lambda = 0$. Find the labeled voltage $V_1$.

\[ V_1 + 5 = I \left( \frac{1}{2} (0.4) (5 - V_1)^2 \right) \]

\[ = \frac{1}{2} (0.4) (5 - V_1^2) \]

\[ \Rightarrow I = \frac{0.45}{0.036 \text{ mA}} \]

\[ T_{es} < V_T \]

\[ V_1 = -5 + 1000 I = -1.4V \]
6. (20 pts) The transistor below has a threshold voltage $V_t = 0.5V$. Furthermore $k_m = 0.4\, \text{mA/}V^2$ and $\lambda = 0$. Show that the transistor operates at the edge of saturation if the following condition is satisfied

$$\frac{W}{L} R_D = 1.5\, \text{k}\Omega$$

**Diagram:**

- $+1.8\, \text{V}$
- $R_D$

**Equations:**

- $k_m = 0.4\, \text{mA/}V^2$
- $V_t = 0.5\, \text{V}$
- $\lambda = 0$

**Saturation Boundary:**

$$V_{GDS} = 5\, \text{V} = R_D I_D$$

$$0.5 = \frac{1}{2} k_m \frac{W}{L} (1.8 - 0.5)^2 R_D$$

$$\Rightarrow \frac{W}{L} R_D = 1.48\, \text{k}\Omega$$
7. (30 pts) For the two amplifiers shown below, draw their small signal equivalent circuit.

- Use the T-model for both transistors in circuit I and neglect the effect of $r_0$.
- For circuit II, using the hybrid-$\pi$ model of the transistor and neglect the effect of $r_0$. Use your small signal equivalent circuit to derive an analytical expression of the input resistance $R_{in}$.
\[ V_{DD} \]

\[ R_L \]

\[ R_1 \]

\[ R_2 \]

\[ +N_{\text{sig}} \]

\[ R_{\text{in}} \]

\[ \pm \]

\[ G \]

\[ R_2 \]

\[ D \]

\[ N_0 \]

\[ R_1 \]

\[ \pm \]

\[ N_{\text{sig}} = N_c \]

\[ +10 \]

\[ \frac{1}{R_1} N_{qs} + N_c \frac{G_{\text{out}} R_2}{R_1} + \frac{G_{\text{in}} N_{qs}}{R_1} \frac{N_0}{R_L} = 0 \]

\[ N_{qs} = N_0 \frac{R_L}{R_1 + R_2} - N_c \]

\[ \frac{G_{\text{out}} R_2}{R_1} + \frac{G_{\text{in}} N_{qs}}{R_1} + \frac{N_0}{R_L} + \frac{N_0 R_1}{R_1 + R_2} = 0 \]

\[ \frac{G_{\text{out}} R_2}{R_1} + \frac{G_{\text{in}} N_{qs}}{R_1} + \frac{N_0}{R_L} = \frac{N_0 R_1}{R_1 + R_2} \]

\[ \frac{G_{\text{out}} R_2}{R_1} + \frac{G_{\text{in}} N_{qs}}{R_1} + \frac{N_0}{R_L} = \frac{N_0 R_1}{R_1 + R_2} \]

\[ \frac{N_0}{R_L} \left[ 1 + \frac{G_{\text{in}} R_1}{R_1 + R_2} + \frac{1}{R_L} \right] = \frac{G_{\text{in}} N_{qs}}{R_1} + N_c \]

\[ N_0 \frac{N_0 R_1}{R_1 + R_2} + \frac{1}{R_L} \]

\[ N_0 \frac{N_0 R_1}{R_1 + R_2} + \frac{1}{R_L} \]
\[ R_i = \frac{N_{\text{in}}}{N_i} \]

\[-g_m N_{\text{in}} s_s \left( \frac{1}{R_L} + \frac{1}{R_1 + R_2} \right) N_0 \]

\[ v_i = -g_m V_{g_s} \]

\[ i_i = \left( \frac{1}{R_L} + \frac{1}{R_1 + R_2} \right) N_0 \]

\[ \frac{i_i}{N_i} = \frac{N_0}{N_i} \left( \frac{1}{R_1 + R_2} + \frac{1}{R_L} \right) \]

\[ v_i = \frac{g_m}{N_i} \left[ \frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_L} \right] \left[ \frac{1}{R_1 + R_2 + \frac{1}{R_L}} \right] \]

\[ R_i^* = \frac{N_{\text{in}}}{v_i} = \left[ \frac{g_m R_1 + 1}{R_1 + R_2} + \frac{1}{R_L} \right] \frac{1}{g_m} \left[ \frac{1}{R_1 + R_2 + \frac{1}{R_L}} \right] \]
8. (30 pts) For the amplifier shown below with 3 terminals X, Y, and Z, show how to connect to which terminal a signal source $v_{sig}$, load resistance $R_L$, and AC ground to build a common-source, common-gate, and common-drain amplifier. Draw the three separate circuits (10 points each).
Common Drain