1. Consider the following phase shift oscillator where the op amps can be assumed to be ideal. Determine an expression for the loop gain $L(s) = \frac{V_o}{V_x}$ in terms of the resistances $R$ and $R_2$, the capacitances $C$ (all the same size) and the $s (=j\omega)$. For the intermediate currents and voltages, use the terminology shown on the figure. Hint: Write the voltage gain $\frac{V_o}{V_x}$ as a set of three ratios, one for each op amp stage. Derive an expression for each voltage ratio from the circuit in terms of the $R$'s and $C$. Determine the oscillation frequency $\omega_o$ in terms of the resistances and capacitance. Find the size of $R_2$ needed relative to $R$ for oscillation.
Putting in $s = j\omega$ and simplifying we get

\[
\frac{v_{02}}{v_{01}} = \frac{-j\omega c_R R_2^2}{1 + 3j\omega c_R - 3c_R^2 R_2^2 - j\omega c_R^3} = \frac{j\omega c_R R_2^2}{(1 - 3\omega^2 c_R^2) + j(3\omega c_R - 3\omega c_R^3)}
\]

To get this to go to zero we need this term to go to zero, so

\[1 - 3\omega^2 c_R^2 = 0 \implies \omega = \frac{1}{\sqrt{3}} c_R\]

Then at $\omega = \omega_0$ we have

\[
\frac{v_{02}}{v_{01}} \bigg|_{\omega = \omega_0} = \frac{\omega_0 c_R R_2^2}{3\omega_0 c_R - \omega_0^3 c_R^3} = \frac{\omega_0^2 c_R R_2^2}{3 - \omega_0^3 c_R^3} = 1
\]

\[
\omega c_R R_2 = 3 - \omega_0^3 c_R^3
\]

\[
\frac{1}{\omega c_R R_2} = 3 - \frac{\omega_0^3 c_R^3}{\omega_2 c_R R_2}
\]

\[
\frac{R_2}{\omega c_R R_2} = 3 - \frac{1}{\omega_0} = \frac{2}{\omega_0}
\]

\[
\frac{R_2}{\omega c_R R_2} = 3 - \frac{1}{\omega_0} = \frac{2}{\omega_0}
\]

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\frac{R_2}{\omega c_R R_2} = 3 - \frac{1}{\omega_0} = \frac{2}{\omega_0}
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ELECTRONICS II
May 27, 2003

I. (20 pts): A second order filter has its poles at

\[ s = \frac{1}{4} \pm j \frac{\sqrt{8}}{4}. \]

The transfer function is zero at \( \omega = 3 \) rad/s and is equal to 4 at DC.

Find the analytical expression for the transfer function \( T(s) \).

\[
T(s) = k \frac{\left( s + j \frac{\sqrt{8}}{4} \right) \left( s - j \frac{\sqrt{8}}{4} \right)}{\left( s + \frac{1}{4} + j \frac{\sqrt{8}}{4} \right) \left( s + \frac{1}{4} - j \frac{\sqrt{8}}{4} \right)}
\]

\[
T(s) = \frac{k \left( s^2 + 9 \right)}{\left( s + \frac{1}{4} \right)^2 + \left( \frac{\sqrt{8}}{4} \right)^2} = k \frac{s^2 + 9}{s^2 + \frac{1}{2} + \frac{8}{16}}
\]

\[
T(s) = k \frac{s^2 + 9}{s^2 + \frac{1}{2} + \frac{9}{16}} = \frac{16k \left( s^2 + 9 \right)}{16s^2 + 8s + 9}
\]

\[\implies T(\omega) = \frac{16k}{16k} = 4\]

\[\implies k = \frac{1}{4}\]

\[\implies T(\omega) = \frac{\left( \omega^2 + 9 \right)}{4 \left( \omega^2 + \frac{1}{2} + \frac{9}{16} \right)}\]
IV. (30 pts): The filter below is said to be of the high-pass notch type, i.e,

\[
T(s) = a_2 \frac{s^2 + \omega_n^2}{s^2 + \alpha \left( \frac{\omega_n}{Q} \right) + \omega_n^2}
\]  

with \(\omega_n\) lesser than \(\omega_o\).

Derive the expression for \(T(s)\) from the circuit diagram and give analytical expressions for \(\omega_n\), \(\omega_o\) and \(Q\) in terms of \(R\), \(L_1\), \(L_2\), and \(C\).

The notch in the transfer function occurs at the angular frequency \(\omega_n\) (see attached table for the second-order high-pass notch filter. For what ratio of \(L_1/L_2\) does \(\omega_n = 0.9 \omega_o\) ?

\[
T(s) = \frac{\frac{R \omega_n L_2}{R + j \omega_n L_2}}{\frac{1}{\omega_n R + \frac{R \omega_n L_2}{C} + \frac{\delta L_1 L_2 R + R L_2}{C}}} = \frac{\frac{\delta R L_2}{\omega_n R + \frac{R \omega_n L_2}{C} + \frac{\delta L_1 L_2 R + R L_2}{C}}}{\frac{1}{\omega_n R + \frac{R \omega_n L_2}{C} + \frac{\delta L_1 L_2 R + R L_2}{C}}}
\]

\[
T(s) = \frac{\frac{\delta L_1 L_2 R + R L_2}{C}}{\frac{1}{\omega_n R + \frac{R \omega_n L_2}{C} + \frac{\delta L_1 L_2 R + R L_2}{C}}}
\]

\[
T(s) = \frac{\frac{\delta^2}{L_1 L_2 R + R L_2}}{\frac{\delta^2 + \frac{1}{L_1 C}}{\delta^2 + \frac{\delta L_1 L_2}{C} + \frac{\delta^2}{(L_1 L_2 C)}}}
\]

\[
\omega_n \text{ notch is at } \omega_n = \frac{1}{L_1 C} \Rightarrow \omega_n = \frac{1}{\sqrt{L_1 C}}
\]

\[
\omega_o = \frac{1}{\sqrt{L_1 L_2 C}} \Rightarrow \omega_o = \left( \frac{L_1 + L_2}{L_2} \right) \omega_n \quad \text{for } \omega_o = (0.9 \omega_n)^2
\]

\[
\frac{L_1}{L_2} = 0.2346
\]
III. (30 pts): Consider the circuit below in which the op-amp is assumed to be ideal (i.e., infinite input resistance looking into the positive and negative terminals). Show that the transfer function for that filter \( T(s) = \frac{V_o}{V_i} \) is of second order and can be expressed as follows

\[
T(s) = \frac{a_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2}
\]  

(3)

Give analytical expressions for \( \omega_0 \) and \( Q \) in terms of \( R_1, R_2, \) and \( C_3, C_4. \)

\[
\omega_0 = \sqrt{\frac{1}{C_3 C_4 R_1 R_2}} \quad \frac{1}{R_1 + \frac{1}{C_4 R_2}}
\]

\[
Q = \frac{\frac{1}{C_3 C_4 R_2}}{\sqrt{C_3 C_4 R_1 R_2}} \quad C_4 \left(\frac{1}{R_1} + \frac{1}{R_2}\right)
\]