Chapter 8 (Basic RL & RC Circuits) pp261-P

Section 8.1 Source free RL circuit.

The circuit equation is a homogeneous first order differential equation,

\[ + \frac{N_R}{R} + \frac{N_L}{L} \]

in which every term is of the first degree in the dependent variable or one of its derivatives.

Natural response: The response depends on the general "nature" of the circuit.

When we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular selection (This is referred to as the forced response).

\[ i(t) \]

\[ R_i + N_L = R_i + L \frac{di}{dt} = 0 \]

\[ \frac{di}{dt} + \frac{R}{L} i = 0 \]

Passive sign convention

Direct approach

\[ \frac{di}{dt} = -\frac{R}{L} i(t) \]

\[ \int_{t_0}^{t} \frac{di}{i} = \int_{t_0}^{t} \frac{R}{L} dt \]

\[ \ln i |_{t_0}^{t} = -\frac{R}{L} (t-t_0) \]
A more general solution approach p.258

Assume \( i(t) = A e^{\alpha_1 t} \)

where \( A, \alpha_1 \) are constants to be determined.

Substitute \( i(t) \) in \( \frac{di}{dt} + \frac{R}{L} i = 0 \)

\[ \Rightarrow A \alpha_1 e^{\alpha_1 t} + A \frac{R}{L} e^{\alpha_1 t} = 0 \]

\[ \Rightarrow (\alpha_1 + \frac{R}{L}) A e^{\alpha_1 t} = 0 \]

This is satisfied if \( A = 0, \alpha_2 = -\infty, \alpha_3 = -\frac{R}{L} \)

Response is zero!

Therefore we must choose \( \alpha_1 = -\frac{R}{L} \)

\[ \Rightarrow i(t) = A e^{-\frac{R}{L} t} \]

But \( i(0) = I_0 = A \)

\[ \Rightarrow i(t) = I_0 e^{-\frac{R}{L} t} \]

\[ (R_1 + \frac{R}{L} = 0 \]

is called a characteristic equation

It characterizes the natural response of the system.
8.2.3 Properties of the exponential response

\[ i(t) = I_0 e^{-\frac{R}{L} t} \]

If \(R > L\), the exponential takes longer to decay.

Initial rate of decay:

\[ \frac{d}{dt} \left( \frac{i(t)}{I_0} \right) = -\frac{R}{L} e^{-\frac{R}{L} t} \]

can be determined from the display on an oscilloscope.

Other interpretation of \(C\):

\[ \frac{i(t)}{I_0} = e^{-t} = 0.3679 \]

Thus, in one time constant, the response has dropped to \(36.8\%\) of its initial value.
\[ \frac{i(t)}{I_0} = 0.1353 \text{ at } t=2 \nu \\
0.04979 \text{ at } t=3 \nu \\
0.01832 \text{ at } t=4 \nu \\
0.006738 \text{ at } t=5 \nu \]

If we are asked "How long does it take for the current to decay to zero?" we can answer "about five time constants."

At that point, the current is less than 1% of its original value.
Energy
Instantaneous power: \( P_R = R \cdot i = \frac{I_0}{R} \cdot e^{-\frac{2Rt}{L}} \)

Total energy turned into heat in resistor is

\[
W_R = \int_0^\infty P_R \, dt = \int_0^\infty \frac{I_0}{R} \cdot e^{-\frac{2Rt}{L}} \, dt
\]

\[
W_R = I_0 R \left( \frac{L}{2R} \right) \left( 1 - e^{-\frac{2Rt}{L}} \right)
\]

\[
W_R = \frac{1}{2} I_0^2
\]

This is the total energy stored initially in the inductor at \( t = 0 \); the current through the inductor has dropped to zero at \( t = \infty \) and the energy must appear as dissipated in the resistor.
Example 8.2 p.260

For the circuit below, find $v(t=0.001ms)$

![Circuit Diagram](image)

$-N + 10i_L + 5\frac{di_L}{dt} = 0$

$i_L = -\frac{N}{40}$

$\frac{5}{40} \frac{dv}{dt} + (\frac{10}{40} + 1)N = 0$

$\frac{dv}{dt} + 10N = 0$

\[\text{When switch is thrown, only the current through the inductor must remain unchanged.}\]
In Fig. 8.5b, \( i_L = \frac{2.4}{10} = 0.24 \) A
(The inductor acts as a short to a DC current)

From Fig. 8.5(c)

\[ V(t) = 40 \left( -2.4 \right) = -96 \text{ V} \]

\[ \frac{dV}{dt} + 10V = 0 \]
\[ \Rightarrow V(t) = A e^{-10t} \]
\[ \Rightarrow \dot{v} + 10 = 0 \]
\[ \Rightarrow \omega = -10 \]
\[ V(t) = Ae^{-10t} \]
\[ \text{At } t = 0^+, \quad V(0^+) = -96 \text{ V} \]
\[ \Rightarrow V(t) = -96e^{-10t} \]

\[ V(0^-) = 24 \text{ V} \]

\[ V(t) \rightarrow \text{JUMP} \]
\[ \text{Zoom in} \]
\[ (\Rightarrow) \]

\[ 24 \text{ V} \]

\[ -96 \text{ V} \]

\[ -13 \text{ V} \]

\[ -10 \text{ V } (0.2) \]

\[ N(t) = -96 e^{-2} \text{ V} \]

\[ = -96 \text{ V } - \text{2 V} \]

\[ = -13 \text{ V} \]
A long time after all connections have been made in the circuit shown above, find $V_x$ if:

- A capacitor is present between $x$ and $y$.
- An inductor is present between $x$ and $y$.

\[ i_x = 2.5\text{A} \]

\[ V_X = \frac{40 \times 15}{6} = 100\text{V} \]

\[ i_x = \frac{12}{2} = 15\text{A} \]

\[ i_x = \frac{15}{6} \]

\[ V_{xy} = \frac{15}{5 + 4 + 1} = 1.5\text{A} \]

\[ \Rightarrow V_{xy} = 40\text{V} \]
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\[ V_L = \frac{1}{2} L i^2 \]
\[ V_C = \frac{1}{2} C V^2 \]

\( V_L \), \( V_C \), \( V \), \( i \), \( V \) across each circuit element.

Current in each circuit element.

Circuit at steady state.

\[ V_{50\Omega} = 16V \]
\[ R_{eq} = \frac{400}{50} = 8 \Omega \]

\[ i = 1.6A \]

-16V across 10Ω resistor.

1.6A flowing through 10Ω and 5H inductance.

\[ N_L = \frac{1}{2} L i^2 \]
\[ N_C = \frac{1}{2} C V^2 \]

\[ = \frac{1}{2} \times 5 \times (1.6)^2 = 6.4 J \]

\[ = \frac{1}{2} \times 10 \times (0.5)^2 = 0.1 J \]
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\[ V_S = 4000e^t \text{ V for } t > 0 \]
\[ i_L(0) = 0.5 \text{ A} \]

At \( t = 0.4 \), find values of energy:
1. Stored in the capacitor.
2. Stored in the inductor.
3. Dissipated by the resistor since \( t = 0 \).

\[ \text{Diagram of circuit} \]

\[ W_R = P_R = \frac{dW_R}{dt} = \int_0^t P_R \, dt = \int_0^t R \cdot i_R^2(t) \, dt = \int_0^t R \cdot \frac{N_S^2(t)}{R} \, dt \]

\[ \frac{W_R(t)}{W_R(0)} = \frac{\int_0^t R \cdot \frac{N_S^2(t)}{R} \, dt}{\int_0^t R \cdot \frac{N_S^2(0)}{R} \, dt} \]

Calculate for \( t = 0.4 \) \( \rightarrow W_R(0.4) = 3.277 \) J

\[ W_C(t) = \frac{1}{2} C \left( \frac{N_S}{R} \right)^2 = \frac{10^{-6}}{2} \left( 4000 \right)^2 \text{ for } t = 0.4 \) \( (20.48 \text{ mJ}) \)

\[ W_L(t) = \frac{1}{2} L \cdot i_L^2(t) = 9.158 \text{ J} \]

\[ i_L(t) = i_L(0) + \int_0^t \text{d}t' \cdot N_S(t') \]

\[ i_L(0.4) = 1.3533 \text{ A} \]
The distinction between $i_L(t)$ and $i_2(t)$.

Assume $t > 0$. At $t = 0^-$, $i_L(0^-) = \frac{-E}{L}$. Therefore, $i_L(t) = i_L(0^-) e^{\frac{-t}{RC}} = i_L(0^+) e^{\frac{-t}{RC}}$.

Using the current divider rule:

$$i_2(t) = \left[\frac{-R_1}{R_1 + R_2}\right] i_L(t)$$

$$i_1(t) = \left[\frac{R_2}{R_1 + R_2}\right] i_L(t)$$

For $t > 0$:

$$i_2(t) = \left(\frac{-R_1}{R_1 + R_2}\right) i_L(0^+) e^{\frac{-t}{RC}}$$

$$i_1(t) = \left(\frac{R_2}{R_1 + R_2}\right) i_L(0^+) e^{\frac{-t}{RC}}$$

All currents have the same time dependence.
Example 8.4 Determine $i_1$ & $i_2$ for $t > 0$

\[ i_L(0^-) = 360 \text{ mA} \]

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\[ \text{leq} = 1 \text{ mH} + \frac{2 \text{ mH}}{2+3} = 2.2 \text{ mH} \]

\[ R_{eq} = \frac{90}{180} + 50 = 110 \text{ \Omega} \]

\[ \hat{v} = \frac{\text{leq}}{R_{eq}} = 20 \text{ A/s} \]

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Natural response is \[ Ke \]

Prior to opening switch

\[ i_L = \frac{18 \text{ V}}{50 \text{ \Omega}} = \frac{18}{50} \text{ A} = 360 \text{ mA} \]

\[ i_L(0^-) = i_L(0^-) \]

\[ i_L(0^-) = 360 \text{ mA} \]

\[ i_L(t = 0) = 360 \text{ mA} \]

\[ i_L(t = 0) = 360 \text{ mA} \]

\[ i(0^-) = \frac{18}{90} = 200 \text{ mA} \]

\[ i(0^-) = \frac{18}{90} = 200 \text{ mA} \]

\[ i(0^+) = \frac{-1(120+60)}{120+60+90} i(0^-) \]

\[ i(0^+) = \frac{-1(120+60)}{120+60+90} i(0^-) \]

\[ i(0^+) = -240 \text{ mA} \]

\[ i(150) = -240 \text{ mA} \]

\[ i(150) = -240 \text{ mA} \]
At $t = 0.15\,\text{s}$, find $i_L, i_1, i_2$

$t < 0$ \quad $i_L(0-) = \frac{8}{10} \cdot \frac{2A}{2} = 1.6A$

$i_1(0-) = \frac{2}{10} \cdot 2A = 0.4A$

$t > 0$ \quad $i_L(0+) = 1.6A$

$t = 0$, switch is closed $\Rightarrow$ $V$ across $R$ is zero.

\[ i(t) = 1.6e^{-\frac{t}{0.2}} \]

\[ C = \frac{L}{R} = \frac{0.4}{2} = 0.2\,\text{s} \]

- $i_L(0.15) = 1.6 \cdot \frac{0.15}{0.2} = 0.756A$
- $i_1(0) + i_1(0.15) = 0$ (inductor is shorted)
- $i_2(0) + i_L(0) = 2A$

\[ i_2(0.15) = 2 - i_L(0.15) = 1.244A \]

$t_L(\infty) \Rightarrow 0$

$t_2(\infty) = 2A$

$i_1(t) = 0$ for $t > 0$

$i_2(t) = 2 - i_L(t) = 2 - 1.6e^{-\frac{t}{0.2}}$