Chapter 6: Bipolar Junction Transistors (BJTs)

pnpn transistor

Emitter  p-type  n-type  Collector

BJT Nodes of operation

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Active Node

e- emitted

e- injected

h+ recombine with e-
h+ injected

I_E  I_B  I_C

- V_{CB} +
Collector Current

电流控制的

电流在集电极端

\( \frac{I_C}{I_s} = e \)

\( I_T = \frac{kT}{q} \)

\( I_s = \text{电流尺度因子} \)

基极电流

\( i_B = \frac{i_C}{B} \frac{N_{BE}}{I_T} \)

\( i_B = \frac{I_s}{B} e \)

\( B = 100-200 \) 是称为共发射极电流增益

发射极电流

\( i_E = i_C + i_B \frac{N_{BE}}{I_T} \)

\( i_E = \frac{B+1}{B} i_C = \frac{B+1}{B} I_s e \)

\( i_C = \frac{B}{B+1} i_E = \alpha i_E \)

\( B = \frac{\alpha}{1-\alpha} \quad \leftrightarrow \quad \alpha = \frac{B}{B+1} \)

\( \alpha = \text{共基电流增益} \)

通常，\( \alpha \approx 1 \rightarrow B \) 太大

是好的要有一个好/传输器

\( i_E = \left( \frac{I_s}{\alpha} \right) e \)
Equivalent Circuit Models

\[ i_e = \frac{I_s}{\alpha} e \]  
\[ i_c = \alpha i_e \]  
\[ N_{BE} \]

\( \alpha \) is Common-Base current gain.

\[ i_b = \frac{I_s}{\beta} e^{\frac{N_{BE}}{T}} \]  
\[ i_c = \sqrt{\beta} i_b \]  
\[ I_s e^{\frac{N_{BE}}{T}} = i_c \]  
\[ V_{BE} \rightarrow V_{CE} \]

\( \beta \) is Common-Emitter Current Gain.

§4.3 The pnp transistor - Active Mode

Base-Emitter junction is forward biased  
Collector-base junction is reverse biased

\[ V_{BE} \rightarrow V_{CE} \]
§4.4 Circuit Symbols and Convention

The arrow points in the direction of normal current flow in the emitter.

Biasing the transistors in the active mode

npn transistor with EB junction. Forward-biased will operate in the active mode. As long as the collector is higher in potential than the base.

pnp transistor will operate in the active mode as long as the potential of the collector is lower than that of the base.
Ex 4.1

\[ V_{BE} = 0.7 \text{V} \quad \text{at} \quad I_C = 1 \text{mA} \]

Given

Design the circuit such that a current of 2mA flows through the collector and \( V_C = +5 \text{V} \).

\[ V_C = +5 \text{V} \quad \Rightarrow \quad R_C = \frac{10 \text{V}}{2 \text{mA}} = 5 \text{k} \Omega \]

\[ i_C = I_S e^{\frac{V_{BE}}{V_T}} \]

\[ V_{BE} = V_T \ln \left( \frac{i_C}{I_S} \right) \]

\[ V_{BE} = V_{BE}^1 + V_T \ln \left( \frac{i_C^2}{i_C^1} \right) \]

\[ i_C^1 = 1 \text{mA} \quad \text{when} \quad V_{BE}^1 = 0.7 \text{V} \]

\[ V_T = 25 \text{mV} \]

\[ \Rightarrow \quad \text{for} \quad i_C^2 = 2 \text{mA} \Rightarrow V_{BE}^2 = 0.727 \text{V} \]

\[ V_B = 0 \text{V} \quad \Rightarrow \quad V_E = -0.727 \text{V} \]

\[ \beta = 100 \quad \Rightarrow \quad \alpha = 0.99 \]

\[ \Rightarrow \quad I_E = \frac{I_C}{\alpha} = 2.02 \text{mA} \quad \left( I_B = 0.02 \text{mA} \right) \]

\[ \Rightarrow \quad R_E = \frac{V_E - (-15)}{I_E} = 4.07 \text{k} \Omega \]
Exercise 4.9

\[ I_E = \frac{10 - 1.7}{5} = 1.66 \text{ mA} \]

\[ \frac{V_B}{100 \text{ k}\Omega} = I_B = \frac{1}{100} = 0.01 \text{ mA} \]

\[ I_C = I_E - I_B = 1.66 - 0.01 = 1.65 \text{ mA} \]

\[ V_C = -10 + (1.65) \times 5 = -1.75 \text{ V} \]

\[ \alpha = \frac{I_C}{I_E} = \frac{1.65}{1.66} = 0.994 \]

\[ B = \frac{I_C}{I_B} = \frac{1.65}{0.01} = 165 \]

(Note: The text indicates that \( B \) is large, close to unity, as expected for a transistor in forward active mode.)
Assume transistor in ACTIVE mode for AC analysis.

Determine all voltages and currents.

\[ V_C = 0.8\text{V} \]
\[ V_B = 5.3\text{V} \]
\[ V_T = 10\text{V} \]

Assumptions:
1. Forward biased base-emitter (\( I_{BE} \))
2. Current through the transistor

\[ I_C = \frac{k I_E}{1 + \frac{k I_E}{I_B}} \]
\[ I_B = \frac{I_E}{1 + \frac{k I_E}{I_B}} \]
\[ V_T = 10\text{V} \]
\[ V_B = 5.3\text{V} \]

Forward bias voltage (\( V_{BE} \))
4.5 Graphical Representation of Transistor Characteristics

(a) 
\[ i_C = I_S e^{\frac{N_{BE}}{V_T}} \]
\[ i_E = I_S e^{\frac{N_{BE}}{V_T}} \]
\[ i_B = \frac{I_S}{\beta} e^{\frac{N_{BE}}{V_T}} \]

Remember

\[ \beta = \frac{q}{1-q} \]

(b) 
Ideal

In practice,

\[ V_{BE} \]

\[ i_C = I_S e^{\frac{N_{BE}}{V_T}} \left( 1 + \frac{N_{CE}}{V_A} \right) \]

\[ V_A = \text{Early Voltage (50-100 V)} \]

In practice

Saturation region
Actic region

Ideally, \[ R_0 = \infty \]

\[ R_0 = \frac{\frac{dV_C}{dN_{CE}}}{N_{BE}} \approx \frac{V_A}{I_C} \]

[\[ N_{BE} = \ldots \]

\[ N_{BE} = \ldots \]

\[ N_{BE} = \ldots \]

\[ \tan \alpha \]
§4.6 Analysis of Transistor Circuits at DC

3. \( I_C = I_E \times \frac{+10V}{2.16mA} \)
   \[ \frac{+10V}{2.16mA} = 4.7k\Omega \]

4. \( V_{BE} = 10V - 1.6 \times 4.7 = 2.48V \) < 6V: cut-off!

\[ \Rightarrow \quad R = \frac{B}{\beta+1} \approx 0.99 \]

\[ \Rightarrow \quad \text{Conclusion} \]

Transistor is NOT in Active Mode!

Saturation Region

5. \( V_{BE} = 6 - 0.7 = 5.3V \)

6. \( 3.3k\Omega \)

7. \( \frac{5.3}{3.3} = 1.6mA \)

8. \( I_C = I_B + I_E \)

\( I_B = 0 \)

Both of the junctions are reverse-biased

cut-off regime.

\( I_E = 0 \)

\[ R_C = 4.7k\Omega \]

\( \text{doesn't conduct} \)

\[ E = 0V \]
Ex.: D4.17

Typical in practice

1. \( V_C \in [50 \text{ to } 150] \)

2. \( R_C \approx \) so that all fabricated transistors will operate in the active mode.

3. What is the range of \( V_C \) that the circuit can exhibit and still be in the active mode?

\[
I_B = \frac{5 - 0.7}{100} = 0.043 \text{ mA} 
\]

\[
V_C = 10 - \beta I_B R_C = 10 - 0.043 \beta R_C \quad (R_C \approx \text{ k}\Omega) 
\]

Design Transistor \( \beta \) so that it is on the verge of saturation for \( \beta = 150 \), i.e., \( V_C = 0.7 + V \)

\[ \Rightarrow \quad R_C = 1.44 \text{ k}\Omega \]

\[ \Rightarrow \quad \beta \leq 150, \text{ the BJT will operate in the active region.} \]

Take \( \beta = 50 \) \[ \Rightarrow \quad V_C = 6.9V \]

\[ \Rightarrow \quad \text{Collector voltage } V_C \in [0.7 \text{ to } 6.9] \text{ volts} \]

\[ \text{for } \beta \in [150 \text{ to } 50] \]

\[ \text{NOT ACCEPTABLE!} \]

We will need to use feedback to control gain of amplifier.
Example 4.4

Determine voltages at all nodes & currents through all branches.

![Circuit Diagram]

**Loop Equation - Very Important.**

\[
V_{bb} = I_B R_{bb} + V_{be} + I_E R_E \\
I_B = \frac{I_E}{\beta + 1} \quad \text{See p117}
\]

\[
I_E = 1.29 \text{ mA}
\]

\[
I_B = \frac{1.29}{101} = 0.0128 \text{ mA}
\]

\[
V_B = (V_{be}) + R_E I_E = 4.5 + V
\]

Active node \(
I_C = \alpha I_E = 0.99 \times 1.29 = 1.28 \text{ mA}
\)

\[
V_C = +15 - I_C R_C = 15 - 1.28 \times 5 = 8.6 \text{ V} > V_B
\]

Active node indeed
Ex 4.8

Find voltages at all nodes & currents through all branches.

\[
\begin{align*}
V_{BB} &= I_B R_{BB} + V_{BE} + R_E I_E \\
I_B &= \frac{I_E}{\beta_2 + 1} \\
I_E &= \frac{V_{BB} - V_{BE}}{R_E + \left[R_{BB}/(\beta_2 + 1)\right]} = 1.29\,mA \\
I_{E2} &= \frac{I_{E2}}{\beta_2} = 0.028\,mA
\end{align*}
\]
Figure 4.24  Linear operation of the transistor under the small-signal condition.

\[ I_C = I_S \frac{v}{V_T} \]

\[ I_E = \frac{I_C}{2} \]

\[ I_B = \frac{I_C}{3} \]

**Fig. 4.23**

Graphically:

Slope = \( g_m = \frac{I_C}{V_T} \)

Typical value:

\( I_C \approx mA \)

\( V_T = 26mV \)

\( g_m \approx 40 \text{ mS} \)

\[ V_{BE} = V_{BE} + V_{be} \]

**Fig. 4.24**
Collector current

\[ I_C = I_S \frac{V_{BE}}{V_T} + \frac{N_{be}}{V_T} \]

\[ I_C = I_S \frac{V_{BE}}{V_T} + N_{be}/V_T \]

\[ I_C = I_S \frac{V_{BE}}{V_T} \left(1 + \frac{N_{be}}{V_T} + \ldots\right) \]

\[ I_C = I_G + \left(\frac{I_C}{I_T}\right) V_{be} = I_G + I_C \]

Define \( g_m \) such that \( I_C = g_m \cdot V_{be} \)

Base Current

\[ I_B = \frac{I_G}{\beta} = \frac{I_G}{\beta} + \frac{1}{\beta} \left(\frac{I_G}{V_T}\right) N_{be} \]

or \[ I_B = \frac{g_m}{\beta} N_{be} \]

Define \[ r_{ie} = \frac{V_T}{I_E} = \frac{\beta}{g_m} \]

Also, \[ r_{ie} = \frac{V_T}{I_B} \]

Emitter Current

\[ i_E = \frac{i_G}{\alpha} = \frac{I_C}{\alpha} + \frac{i_C}{\alpha} = \frac{I_C}{\alpha} + \left(\frac{g_m}{\alpha}\right) N_{be} \]

\[ i_E = \frac{i_C}{\alpha} = \frac{I_E}{V_T} N_{be} = \left(\frac{I_C}{\alpha} \frac{1}{V_T}\right) N_{be} \]

Define \[ r_E = \frac{N_{be}}{i_e} = \frac{V_T}{I_E} = \frac{\alpha}{g_m} = \frac{1}{g_m} \]

\[ N_{be} = i_B r_{ie} = i_e r_e \Rightarrow \beta = \left(\frac{i_E}{i_B}\right) r_e = (\beta + 1) r_e \]
Common Emitter Configuration

\[ V_C = V_{CC} - R_C i_C \]
\[ = V_{CC} - (I_C + i_C) R_C \]
\[ = (V_{CC} - R_C I_C) - i_C R_C \]
\[ = V_C - i_C R_C \]
\[ \frac{V_C}{V_C} \]

\[ \rightarrow \quad V_C = -i_C R_C = -g_m \overline{V_{be}} R_C \]
\[ V_C = (-g_m R_C) \overline{V_{be}} \]

\[ \Rightarrow \quad \text{Voltage Gain} \equiv \frac{V_C}{V_{be}} = -g_m R_C \]

Typical Value

\[ g_m \approx 40 \text{ mS} \]
\[ R_C \approx \text{few k\ohms} \]
\[ \overline{V_{be}} \rightarrow \frac{V_C}{V_{be}} \approx \text{few hundred} \]
§4.8 Small Signal Equivalent Circuit.

\[ i_b = \frac{N_{be}}{r_{e}} \]

\[ i_c = \frac{q_m N_{be}}{r_{e}} \]

\[ i_e = \frac{N_{be}}{r_{e}} \]

\[ r_e = \frac{v_t}{i_e} \quad r_{e1} = \frac{v_t}{i_{Be}} \quad g_m = \frac{I_c}{V_t} \]

Equivalent Transistor Model for small signal analysis.

Hybrid-\(T\) Model: for AC Spice simulations.

\[ i_e = \frac{N_{be}}{r_{e1}} + q_m N_{be} = \frac{N_{be}}{r_{e1}} \left(1 + q_m \frac{r_{e1}}{r_{e}}\right) \]

\[ i_e = \frac{N_{be}}{r_{e1}} \left(1 + \beta_3\right) = N_{be} / \left(\frac{r_{e1}}{1 + \beta_3}\right) = N_{be} / r_e \]

Also \[ q_m N_{be} = q_m (i_b r_{e1}) = (q_m r_{e1}) i_b = \beta_3 i_b \]
The T-model

\[ i_c, i_e \text{ are correct. How about } i_b? \]

\[ i_b = \frac{V_{be}}{r_e} - g_m V_{be} = \frac{V_{be}}{r_e} (1 - g_m r_e) \]

\[ i_b = \frac{V_{be}}{r_e} (1 - \alpha) = \frac{V_{be}}{r_e} \left(1 - \frac{1}{1+\beta}\right) \]

\[ i_b = \frac{V_{be}}{(\beta+1) r_e} = \frac{V_{be}}{r_e} \]

Procedure:  
1. DC operating point is determined
   \[ \rightarrow \text{calculate model parameters} \]
   \[ g_m, r_e, r_c \]

2. DC Sources are removed
   \[ \rightarrow \text{Proceed with AC analysis} \]
   \[ \text{using Hybrid-}T \text{ model} \]
   \[ \text{\rightarrow voltage gain of transistor} \]
Problem 4.51 p 263

Very light P3

\[ V_C = 5 - 2 \times 1 = +3 \text{V} \]

\[ q_m = \frac{I_C}{V_I} = \frac{1 \text{mA}}{0.025 \text{V}} = 40 \text{mA/V} \]

Typical

\[ V_{be} = -V_i \rightarrow V_C = -q_m V_{be} \times 2k\Omega \]

\[ V_C = q_m V_i \times 2k\Omega \]

\[ \frac{V_C}{V_i} = 40 \text{mA} \times 2k\Omega = 80 \text{V/V} \]

Only are \( V_C \) & \( V_i \) in phase?
Augmenting the Hybrid-\(H\) model to account for the Early Effect.

\[ V_{\text{be}} = V_{\text{be}} \]

\[ r_0 \text{ is then in } \parallel \text{ with } R_R \]

\[ V_T = -\frac{g_m}{r_0} (R_R \parallel r_0) \quad [r_0 = \frac{V_T}{I_C}] \]

In general, \( r_0 \) can be neglected if \( \frac{V_T}{I_C} > 10 R_R \).

Exercise 4.24

**Exercise:**

**Q:** What are \( g_m \), \( r_0 \)?

\[ \text{DC Analysis} \]

- \( I_C = \alpha I_E = 0.99 \text{mA} \)
- \( I_B = \frac{I_E}{P_{3+1}} \geq 0.01 \text{mA} \)
- \( V_B = -0.01 \times 10 = -0.1 \text{V} \)
- \( I = 1 \text{mA} \)
- \( V_C = 10 - 0.99 \times 8 = +2.7 \text{V} \)
- \( V_E = -0.1 - 0.7 = -0.8 \text{V} \)

\[ g_m = \frac{I_C}{V_T} = \frac{0.99 \text{mA}}{25 \text{mV}} = 40 \text{mV/A} \]
\[ r_H = \frac{1}{g_m} = \frac{100}{40 \text{mV/A}} = 2.5 \text{k}\Omega \]
\[ r_0 = \frac{V_T}{I_C} = \frac{100 \text{V}}{0.99 \text{mA}} = 100 \text{k}\Omega \]

Neglect \( V_A \) in DC analysis, but not in AC analysis.
\[ z \text{ is connected to ground} \rightarrow \text{common emitter configuration} \]

- To signal source \( V_S \), \( R_S = 2k\Omega \)
- \( y \) to a \( 8k\Omega \) load
- Use hybrid \( H \)-model and draw small-signal equivalent circuit of transistor.
- Calculate \( \frac{V_Y}{V_S} \)
- If \( r_o \) is neglected, what is the error in estimating the gain.

\[
\frac{V_Y}{V_S} = \frac{(10/2.5)}{(10/2.5) + 2} = \frac{2}{2+2} = 0.5
\]

\[
V_Y = -g_m V_T \left( \frac{r_o}{1.8/8} \right)
\]

\[
V_Y = -40 \left( \frac{100/1.8/8} {8/8} \right) V_T = -153.8 V_T
\]

\[ \Rightarrow \frac{V_Y}{V_S} = -7.7 \text{ V/V} \]

Neglecting \( r_o \), \[ \frac{V_Y}{V_S} = -0.5 \times 40 \left( \frac{8/8} {8/8} \right) = -80 \text{ V/V} \]

- Error of 3.9% only
Example 4.9

Assume $R_2 = 100$. What is the voltage gain of the following transistor amplifier?

\[
V_{CC} = +10V
\]

\[
R_C = 3k\Omega
\]

\[
R_B = 100k\Omega
\]

\[
V_{BB} = 3V
\]

**DC Analysis**

1. Apply the voltage $+10V$ to the circuit.
2. The voltage at node $C$ is $3.1V$.
3. Current analysis:
   - Current $I_C = 2.3mA$.
   - Current $I_B = 0.023mA$.

**AC Analysis**

Draw Hybrid-$Y$ first!

\[
R_{BB} = 100\Omega
\]

\[
R_C = 3k\Omega
\]

\[
g_m = \frac{I_C}{V_T} = \frac{2.3mA}{25mV} = 92\text{ mA/V} \quad \Rightarrow \quad R_T = \frac{R}{g_m} = \frac{100}{92} = 1.09k\Omega
\]

\[
V_{be} = V_T \left( \frac{1}{\frac{1}{R_T} + R_{BB}} \right) = V_T \left( \frac{1.09}{101.09} \right) = 0.011V
\]

\[
V_o = -g_m V_{be} R_C = -92 \times 0.011V \times 3 = -3.04V
\]

\[
\Rightarrow \text{gain} = \frac{V_o}{V_i} = -3.04
\]
Figure 4.29  Signal waveforms.

\[ N_{be} = \frac{N_i}{\sqrt{\frac{r_\mu}{r_\mu + R_{be}}}} \Rightarrow V_{pe} = 10 \text{ mV amplitude} \]

\[ V_{pe} = 0.91 \text{ mV} \]

\[ r_e = 0.91 \times \text{gain} = 2.77 \text{ V} \]

(a)

(b) \[ i_b = 0.008 \text{ mA} \]

(c) \[ V_{be} = 8.6 \text{ mV} \]

(d) \[ i_c = 2.3 \text{ mA} \]

(e) \[ V_c = 2.43 \text{ V} \]

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