**Thevenin's Theorem**

**Statement**

1. Given any linear circuit, rearrange it in the form of two networks A and B, connected by two wires.

2. Disconnect network B \( \Rightarrow \) replace it by infinite load. Define a voltage \( V_{oc} \) as the voltage appearing across the terminals of network A. \( \Rightarrow V_{TH} \)

3. **ZERO OUT** every independent source in network A to form an inductive network (DO NOT TOUCH DEPENDENT SOURCES) \( \Rightarrow R_{TH} \)

4. Connect an independent voltage source \( V_{oc} \) in series with the inductive network from part 3. \( \Rightarrow \frac{V_{TH}}{R_{TH}} \)

5. Connect network from step 4 back to network B. All currents and voltages in B will be the same as what would have been found without the Thevenin equivalent circuit for network A.
Using Thevenin Theorem

\[ V_{Th} = \frac{6}{3+6} \]
\[ I_Z = \frac{2}{3} \]
\[ I_Z = 8V \]

\[ R_{Th} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2 \Omega \]
\[ R_{Th} = 9 \Omega \]
1. Given any linear circuit, rearrange it in the form of two networks (A & B). A is the network to be simplified.

2. Disconnect network B, and short the terminals of A. Call $i_{sc}$ the current flowing through the terminals of A.

3. "Zero out" every independent source in network A to form an inactive network. Leave dependent sources unchanged.

4. Connect an independent current source with value $i_{sc}$ in parallel with the inactive network.

5. Connect network B to the two terminals of network B.

Connection: Thévenin & Norton

\[ \frac{V_{th}}{R_{th}} = I_{sc} = \frac{V_{oc}}{R_{th}} \]

\[ V_{oc} = V_{th} \]

\[ V_{th} \quad P \quad R_{th} \]

\[ I_{sc} \quad \frac{1}{R_{th}} \]
Back to Example 5.6  (Novitton Equivalent?)

\[ R_{eq} = \frac{7.6}{13} = \frac{42}{13} \, \Omega \]

\[ I_{N} = \frac{42}{81} - \frac{12}{81} A = \frac{30}{9} A \]

\[ R_{N} = 7 + \frac{18}{9} = 9 \, \Omega \]

\[ V_{Th} = I_{N} R_{Th} = 8 \, V \]
Practice problem 56  P. 142

Use Thévenin equivalent to find current through 2 \( \Omega \) resistor in circuit below.

\[ V_{Th} = \left( \frac{4}{4+10} \right) 9 \text{ V} = \frac{4}{14} 9 \text{ V} = 2.571 \text{ V} \]

\[ R_{Th} = \frac{5 \Omega}{4 \Omega + 10} \]

\[ 5 + \left( \frac{4 \times 10}{4 + 10} \right) = 5 + \frac{40}{14} = 7.857 \Omega \]

\[ i_L = \frac{V_{Th}}{R_{Th} + 2} = \frac{2.571 \text{ V}}{7.857 \Omega + 2} = 0.2608 \text{ mA} \]
Example 5.4  p.141

\[ i_L ? \]

\[ V_{Th}, R_{Th} \]

\[ V_{Th} ? \]

\[ i = 0 ! \]

\[ R_L = \infty \rightarrow V_{ab} = V_{Th} = V_{oc} \]

\[ V_{Th} = V_{oc} = \left[ \frac{6}{3+6} \right] 	imes 12 = 8V \]

\[ R_{Th} ? \] Replace 12V supply with a short circuit

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7Ω in series with parallel combination of 3Ω and 6Ω

\[ R_{eq} = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega \]

\[ \rightarrow R_{Th} = 7 \Omega + 2 \Omega = 9 \Omega \]

\[ 8V = V_{Th} \]

\[ i_L = \frac{8V}{16 \Omega} = 0.5A \]
Example 5.8 (P 144)

Find the Thévenin and Norton equivalent circuits for the network fed by the 1 kΩ load.

\[ R_{th} = 5 \Omega \]

\[ V_{osc} = V_{Th} \]

Set \( R_L = \infty \)

\[ i = 0 A \]

\[ V_{osc} = 4 V \]

\[ V_{osc} = 2 k\Omega \times 2 mA = 4 V \]

\[ V_{osc} = 8 V \]
I_L can now be found easily using current divider rule.

To practice problem 5.7.

When dependent sources are present.
\[ i_{10} = \frac{7.5 - 10}{2.5 + 10} = \frac{-2.5}{12.5} \]

\[ = -0.2 \text{ A} \]
EXAMPLE

To find \( V_T \):

\[ V_T = (V_{AB})_{oc} \]

\[ V_T = \frac{10}{10+5} \cdot 10 \]

\[ V_T = 6.67 \text{ V} \]

To find \( R_T \):

\[ R_T = 20 + \frac{(5)(10)}{5+10} \]

\[ R_T = 23.3 \Omega \]

\[ i = \frac{6.67}{4+23.3} \]

\[ i = 0.244 \text{ A} \]
(Previous) EXAMPLE

\[ R_n = R_T = 23.3 \, \Omega \text{ as before.} \]

To find \( i_n \):

\[ i_n = (i_{AB})_{SC} \]

\[ i_n = \left( \frac{3.33}{3.33 + 20} \right) \cdot 2 = 0.286 \, A \]

\[ i_n = 0.286 \, A \]

(And check)

\[ v_T = R_T \cdot i_n \]
EXAMPLE

Use source transformations to replace network (all except 10Ω resistance) by Thevenin equivalent

\[ R_T = \frac{9(1.82)}{9 + 1.82} = 1.51 \Omega \]

\[ V_T = 25 - 9 \left( \frac{20.45}{10.82} \right) = 8.00 \text{ V} \]

2 ways:
1. Calculate current through loop
2. Principle of superposition

\[ i = \frac{8.00}{1.51} = 5.29 \text{ A} \]
When dependent sources are present p.145

If network A contains a dependent source, we must ensure that the controlling variable CANNOT BE in network B.

\[ V_x = V_{oc} = V_{TH} \]
\[ i_1 = 4 + 2 \times 10^{-3} \left( -\frac{V_x}{4000} \right) + 3 \times 10^{-3} (0) + V_c = 0 \]

\[ \Rightarrow V_x = V_{oc} = 8 \text{ V} \]

\[ R_{TH} = 4 \text{ V} \rightarrow 0 \text{ V} \text{ but we do not know what } \frac{V_x}{4000} \text{ is} \]

Let's go to Norton equivalent instead. Find \( I_{sc} \) if we short-circuit terminals of network A.

\[ \Rightarrow V_x = 0 \Rightarrow \frac{V_x}{4000} = 0 \text{ or } \text{zero} \]

\[ \Rightarrow \text{Network for } I_{sc} \]

\[ I_{sc} = \frac{4 \text{ V}}{5 \text{ k}\Omega} = 0.8 \text{ mA} \]

\[ R_{TH} = \frac{V_{oc}}{I_{sc}} = \frac{8}{0.8 \times 10^{-3}} = 10 \text{ k}\Omega \]
So the Thevenin eq. is

Practice 5.8

\[ V = 100V \]

\[ 20kΩ \]

\[ 0.01V \]
EXAMPLE

To find $V_T$:

$\frac{2i_4}{2+9} = 0.909 \text{ A}$

$V_T = (V_{AB})_{oc} = -2i_4 + 5i_4 = 3i_4 = 2.73 \text{ V}$

To find $i_n$:

$V_{CB} = 2i_4$
\[ U_{CB} = 2i_4 = 10 - 6i_4 \]
\[ 8i_4 = 10 \]
\[ i_4 = \frac{10}{8} = 1.25 \]
\[ i_n = i_4 - \frac{U_{CB}}{5} = i_4 - \frac{2i_4}{5} = \frac{3}{5} i_4 \]

\[ = 0.75 \text{ A} \]

To find \( R_T \):

\[ R_T = \frac{U_T}{i_n} = \frac{2.73}{0.75} = 3.64 \Omega \]

Or find \( R_T \) directly:
\[ V_{BC} = -2i_4 - 4i_4 - 2i_4 = -8i_4 \]
\[ = -2i_4 + 5i_5 = -2i_4 + 5(i_\infty + i_4) \]
\[ -8i_4 = -2i_4 + 5(i_\infty + i_4) \]
\[ -11i_4 = 5i_\infty \]
\[ i_4 = \frac{-5}{11} i_\infty \]
\[ V_{BC} = -8i_4 = \frac{40}{11} i_\infty \]
\[ R_T = \frac{V_{BC}}{i_\infty} = \frac{40}{11} = 3.64 \Omega \]
Maximum power transfer theorem

\[ P_L = i_L^2 R_L \]

For what value of \( R_L \) is \( P_L \) maximum?

\[ i_L = \frac{V_T}{R_T + R_L} \]

\[ P_L = i_L^2 R_L = \frac{V_T^2 R_L}{(R_T + R_L)^2} \]

\[ \frac{dP_L}{dR_L} = 0 = V_T^2 \frac{(R_T + R_L)^2 - R_L^2(2)(R_T + R_L)}{(R_T + R_L)^4} \]

\[ 0 = R_T^2 + 2 R_T R_L + R_L^2 - 2 R_L R_T - 2 R_L^2 \]

\[ = R_T^2 - R_L^2 \]

\[ R_L^2 = R_T^2 \]

\[ \rightarrow R_L = R_T \]

\[ P_{L_{\text{max}}} = i_L^2 R_L = \frac{V_T^2}{(R_T + R_T)^2} R_T = \frac{V_T^2}{4R_T} \]
Note efficiency of power transfer to $R_L$:

$$\eta = \frac{P_L}{P_{\text{in}}} = \frac{i_L^2 R_L}{i_L^2 (R_L + R_T)} = \frac{R_L}{R_L + R_T}$$

Note other maximum power transfer questions:

- For max power to $R_2$:
  $$R_2 = R_1 + R_T$$

- For max power to $R_1$:
  $$R_2 = 0$$

- For max power to $R_L = R_1 + R_2$:
  $$R_1 + R_2 = R_T$$
  $$R_2 = R_T - R_1$$

Maximum power to $R_2$? $R_1$? $R_L$? $R_L = \frac{R_1 R_2}{R_1 + R_2}$?
Max. Power transfer?

**Practice problem 5.10 p 152**

\[ R_{Th} = \frac{2k\Omega}{2k\Omega} = 1 \, k\Omega \]

\[ V_{Th} \]

\[ V_{ab} = 40 + V_{oc} \Rightarrow V_{Th} = V_{ab} - 40V \]

\[ 2k\Omega \cdot i + 50 + 2k\Omega \cdot i = 0 \]

\[ 4k\Omega \cdot i = -50 \]

\[ i = -\frac{50}{4} \, mA = -12.5 \, mA \]

\[ V_{ab} = 30 + 2k\Omega \cdot i = 30 - 25 = 5V \]

\[ \Rightarrow V_{Th} = -35V \]

\[ P_{max} = \frac{V_{Th}^2}{4R_{Th}} \]

\[ P_{max} = \frac{(-35)^2}{4 \times 1 \, k\Omega} = \frac{1225}{4} = 306 \, mW \]
Practice 5.10  P152

\[ \text{R}_{\text{out}} = 3 \, \text{k}\Omega, \text{ find power delivered to it.} \]

\[ 2 \, \text{k}\Omega \, i_1 + 50 + 2 \, \text{k}\Omega (i_1 - i_2) = 0 \]
\[ 2 \, \text{k}\Omega (i_2 - i_1) - 30 + 40 + \text{R}_{\text{out}} \, i_2 = 0 \]

\[ 4 \, \text{k}\Omega \, i_1 - 2 \, \text{k}\Omega \, i_2 + 50 = 0 \]
\[ -2 \, \text{k}\Omega \, i_1 + (\text{R}_{\text{out}} + 2 \, \text{k}\Omega) \, i_2 + 10 = 0 \]

\[ \begin{align*}
(\times 2) & : & -2 \, \text{k}\Omega \, i_1 + 5 \, \text{k}\Omega \, i_2 + 10 = 0 \\
\rightarrow & : & -4 \, \text{k}\Omega \, i_1 + 10 \, \text{k}\Omega \, i_2 + 20 = 0 \\
\end{align*} \]

\[ 8 \, \text{k}\Omega \, i_2 + 40 = 0 \quad \text{(sum)} \]
\[ i_2 = -\frac{40}{8 \, \text{k}\Omega} \quad \text{A} \]

\[ \text{P}_{\text{out}} = \frac{3 \, \text{k}\Omega \cdot 40^2}{64 \, \text{k}\Omega^2} = \frac{3 \times 40^2}{64} \, \text{mW} \]

\[ \text{P}_{\text{max}} = \frac{210 \, (40)}{64} = 230 \, \text{mW} \]

\[ \text{Fund on previous page} \]

\[ P_L = 306 \, \text{mW} \]
What are values of \( R_{out} \) will have exactly 20 mW delivered to them?

\[
\begin{align*}
4R_{s2}i_1 - 2R_{s2}i_2 + 50 &= 0 \\
-2R_{s2}i_1 + (R_{out} + 2R_{s2})i_2 + 10 &= 0 \\
-4R_{s2}i_1 + (2R_{out} + 4R_{s2})i_2 + 20 &= 0
\end{align*}
\]

Add

\[
-\left( R_{out} + 1R_{s2} \right)i_2 = -70
\]

\[
i_2 = -\frac{35}{(R_{out} + 1R_{s2})}
\]

\[
R_{out} = \frac{R_{out} - \frac{35^2}{(R_{out} + 1R_{s2})^2}}{20 \cdot 10^{-3} (R_{out} + 1R_{s2})^2}
\]

\[
35^2 R_{out} = 20 \cdot 10^{-3} (R_{out} + 1R_{s2})^2
\]

\[
R_{out}^2 + 2R_{s2} R_{out} + 10^6 = \frac{35^2}{20} \cdot 10^{-3} R_{out}
\]

\[\Rightarrow 2 \text{ roots} \] \[
\begin{align*}
R_{out} &= 59.2 \Omega \\
&\approx 16.88 \Omega
\end{align*}
\]
Determine the Thévenin equivalent of the circuit below.

\[ V_{th} = 0 \] (no independent sources)

\[ V_{be} = -2V_x \]

\[ V_{bc} = -2V_x \]

\[ V_{eb} = V_x \]

\[ \frac{1}{2}V_x \]

\[ i = \frac{V_x}{2} + \frac{V_x}{4} = \left(\frac{1}{2} + \frac{1}{4}\right)V_x \]

\[ R_{th} = \frac{V_x}{i} = \frac{1}{\left(\frac{3}{4}\right)} = \frac{4}{3} = 1.33 \Omega \]

Norton equivalent in parallel with 1.33\( \Omega \).

\[ \frac{P}{16} \]

Any value what Beyer may be selected for \( R_L \) which is the maximum power that could be dissipated in \( R_L \)?

\[ R_{th} = \frac{V_{eb}}{i} \]

\[ 8 \Omega \] in parallel with \( 12 \Omega \).

\[ \frac{1}{R} = \frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24} \]

\[ R = \frac{24}{5} = 4.8 \Omega \]

5\( \Omega \) and 6\( \Omega \) are series.

\[ R_1 = 5\Omega + 6\Omega \]

\[ R_{th} = 15 + 8 \Omega \]

\[ R_{th} = 16 \Omega \]
\[ V_1 = 20 \left( \frac{8}{8+12} \right) = \frac{20 \cdot 8}{20} = 8 \text{V} \]

\[ V_2 = -2 \times 6 = -12 \text{V} \]

\[ V_{Th} = V_1 - V_2 = 8 \text{V} + 12 \text{V} = 20 \text{V} \]

\[ P_{\text{max}} = \frac{V_{Th}^2}{4 R_{Th}} = \frac{(20)^2}{4 \times 16} = \frac{25}{4} = 6.25 \text{W} \]

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**Stage 1**

**Stage 2**

Select \( R_1 \) so that the maximum power is transferred from Stage 1 to Stage 2.

\[ R_{Th} = R_1 = 8 \text{k}\Omega \]

\[ V_{Th} = (4 \times 10^{-3})(7 \times 10^3) = 280 \text{V} \]