Chapter II: Introduction to electronics p.45 Sections 14-16. 

1.4 Amplifiers

- Transducers provide signals that are weak (mV, nV) and with little energy.
- Amplifiers must preserve the details of the signal waveform.
- Amplifiers provide gain.

\[ v_o(t) = A v_i(t) \]

Linear amplifier gain

If relation is not linear ⇒ distortion is said to be introduced in the signal.

Power amplifier: provides only a modest amount of voltage gain but substantial current gain (home stereo system: you need enough power to drive the loudspeaker).

Amplifier Circuit Symbol: two-port network.

Voltage Gain

\[ A_V = \frac{v_o}{v_i} \]

Power Gain (an amplifier increases the signal power, contrary to a transformer for instance).

\[ A_P = \frac{P_o}{P_i} \]
\[ A_P = \frac{V_{00}}{V_{11}} \]

Current Gain: \( A_i = \frac{I_o}{I_H} \)

For historical reasons
- Volt. gain \( \text{in} \ dB = 20 \log |A_V| \ dB \)
- Cur. gain \( \text{in} \ dB = 20 \log |A_i| \ dB \)
- Power \( \text{in} \ dB = 10 \log |A_P| \ dB \)

Negative gain \( A_V \) means that there is a 180° phase difference between input and output signals.

**Amplifier Power Supplies**
- Power delivered to load is > power drawn from the signal
- Source of additional power? Amplifiers need DC power supplies

**AC signals (sine waves)**

\[ P_L = V_{rms} I_{rms} = \frac{V_{ampl}}{\sqrt{2}} \cdot \frac{I_{ampl}}{\sqrt{2}} \]

\[ P_{out} < P_L \rightarrow A_P = \frac{P_L}{E_i} \]
DC Power delivered to amplifier

\[ P_{dc} = V_1 I_1 + V_2 I_2 \]

Power dissipated in amplifier circuit:

\[ P_{dissipated} = P_i \]

\[ P_i = P_L = P_{dissipated} \]

Balance Eq.

\[ P_{dc} + P_i = P_L + P_{dissipated} \]

Efficiency:

\[ \eta = \frac{P_L}{P_{dc}} \times 100 \]
Consider an amplifier operating from ±10-V power supplies. It is fed with a sinusoidal voltage having 1 V peak and delivers a sinusoidal voltage output of 9 V peak to a 1-kΩ load. The amplifier draws a current of 9.5 mA from each of its two power supplies. The input current of the amplifier is found to be sinusoidal with 0.1 mA peak. Find the voltage gain, the current gain, the power gain, the power drawn from the dc supplies, the power dissipated in the amplifier, and the amplifier efficiency.

**Solution**

\[ A_v = \frac{9}{1} = 9 \text{ V/V} \]

or

\[ A_v = 20 \log 9 \approx 19.1 \text{ dB} \]

\[ i_o = \frac{9 \text{ V}}{1 \text{ kΩ}} = 9 \text{ mA} \]

\[ A_i = \frac{i_o}{i_i} = \frac{9}{0.1} = 90 \text{ A/A} \]

or

\[ A_i = 20 \log 90 = 39.1 \text{ dB} \]

\[ P_L = V_{\text{rms}} I_{\text{rms}} = \frac{9 \cdot 9}{\sqrt{2} \cdot \sqrt{2}} = 40.5 \text{ mW} \]

\[ P_I = V_{\text{rms}} I_{\text{rms}} = \frac{1 \cdot 0.1}{\sqrt{2} \cdot \sqrt{2}} = 0.05 \text{ mW} \]

\[ A_p = \frac{P_L}{P_I} = \frac{40.5}{0.05} = 810 \text{ W/W} \]

or

\[ A_p = 10 \log 810 = 29.1 \text{ dB} \]

\[ P_{\text{dc}} = 10 \times 9.5 + 10 \times 9.5 = 190 \text{ mW} \]

\[ P_{\text{dissipated}} = P_{\text{dc}} + P_I - P_L \]

\[ = 190 + 0.05 - 40.5 = 149.6 \text{ mW} \]

\[ \eta = \frac{P_L}{P_{\text{dc}}} \times 100 = 21.3\% \]
Figure 1.11  A linear amplifier with output saturation.

Output peaks clipped due to saturation

Input waveforms

Output waveforms

Fig. 1.11
Figure 1.12 Obtaining linear operation from a nonlinear amplifier.
Example: \( n_0(v) \)

\[ \begin{array}{c}
10 \\
5 \\
0.3 \\
0.673 \\
0.690 \\
\end{array} \]

\[ \begin{array}{c}
v_0 = 10 \Rightarrow n_0 = 5 \\
v_0 = 5 \Rightarrow n_0 = 10 \\
v_0 = 5 \Rightarrow n_0 = 5 \\
\end{array} \]

\( L_+ = 10 \) \( \Rightarrow \) \( n_+ = 0.69 \)

\( L_- = 0.3 \) \( \Rightarrow \) \( n_- = 0.673 \)

\( n_0 \) \( \Rightarrow \) \( n_+ = 0.69 \)

\( n_0 \) \( \Rightarrow \) \( n_- = 0.673 \)

\[ A_v = \frac{dn_0}{dn_+} = -200 \text{V/V} \]

\( n_+ = 0.69 \)

Notation:
- Instantaneous quantities: \( i_A(t), n_C(t) \)
- Direct-current (DC) \( i_A, v_A \)
- Incremental signal quantities: \( i_a(t), v_a(t) \)
Circuit Model for Amplifiers

$R_i \approx$ input resistance (amplifier draws input current from source)

$R_o \approx$ output resistance (accounts for change in output voltage as the amplifier supplies output current to load)

Voltage amplifier with input signal source & load

\[
N_o = A_v \frac{N_r}{N_o} = \left( \frac{R_L}{R_L + R_o} \right)
\]

\[
N_L = \left( \frac{R_i}{R_i + R_s} \right) N_s
\]

\[
\frac{N_o}{N_i} = A_v = A_v \left( \frac{R_L}{R_L + R_o} \right)
\]

- To not kill $A_v$, $R_o$ should be much smaller than $R_L$
- Ideal voltage amplifier is one with $R_o = 0$
- For $R_L = \infty$, $A_v = A_v \frac{N_r}{N_o} \approx$ open circuit voltage gain.
- Design amplifier such that $R_i \gg R_s$
- Ideal voltage amplifier has $R_i = \infty$ (current & power gains become infinite).
Multiple Stages

See Example 1.3 in book p. 19

\[ \frac{N_L}{N_{E_1}} = A_n = A_{n_1} A_{n_2} A_{n_3} \quad \text{and} \quad \frac{N_L}{N_{E_3}} = \frac{N_{E_2}}{N_{E_4}} = A_n \frac{N_{E_2}}{N_{E_3}} \]

\[ A_{L_i} = \frac{i_o}{i_1} = \frac{N_L/R_L}{N_{E_i}/R_i} \]

\[ A_P = \frac{P_o}{P_i} = \frac{N_L}{N_{E_1}} i_o / i_1 = A_n A_{L_i} \]

\[ \frac{N_{E_1}}{N_{E_3}} = \frac{1 M\Omega}{1 M\Omega + 100 \text{ k}\Omega} = 0.909 \]

\[ A_{n_1} = \frac{N_{E_2}}{N_{E_1}} = 10 \left( \frac{100 \text{ k}\Omega}{100 \text{ k}\Omega + 1 \text{ k}\Omega} \right) = 9.9 \]

\[ A_{n_2} = \frac{N_{E_3}}{N_{E_2}} = 100 \left( \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 1 \text{ k}\Omega} \right) = 90.9 \]

\[ A_{n_3} = \frac{N_L}{N_{E_3}} = 1 \left( \frac{100}{100 + 10} \right) = 0.909 \approx 1 \quad \text{Stage 3 is referred as Buffer Amplifier} \]
\[ A_{V3} = \frac{N_L}{N_S} = A_{N1} A_{N2} A_{N3} = 8.18 \, V/V \quad (58.3 \, \text{dB}) \]

\[ \Rightarrow \frac{N_L}{N_S} = A_{N1} \left( \frac{N_S}{N_{11}} \right) = 8.18 \left( \frac{0.909}{} \right) = 7.43 \, \text{V/V} \]

**Current gain**
\[ A_i = \frac{i_o}{i_i} = \frac{N_L}{100 \Omega \cdot N_{11}/10 \Omega} = 10^4 \, A_{N1} = 8.18 \cdot 10^6 \, \text{A/A} \]

**Power gain**
\[ A_p = \frac{P_L}{P_i} = \frac{N_L^2 \cdot i_o}{N_{11}^2} = A_{N1} A_i = 66.9 \cdot 10^8 \, \text{W/W} \quad (98.3 \, \text{dB}) \]

**Exercise:** Show that: \[ A_P (\text{dB}) = \frac{1}{2} \left[ A_{N1} (\text{dB}) + A_i (\text{dB}) \right] \]

---

**Other Amplifier Types**

**Current amplifier**

![Current Amplifier Circuit Diagram]

**Current divider rule:**
\[ i_o = A_{iS} i_i \left( \frac{R_o}{R_o + R_L} \right) \]

**Current gain of loaded amplifier:**
\[ A_i = \frac{i_o}{i_i} = A_{iS} \left( \frac{R_o}{R_o + R_L} \right) \]

- **Best design:** \( R_o \gg R_L \)
- **Ideal current amplifier:** \( R_i = 0 \)

**Ideal current amplifier**

- \( i_i = i_i \left( \frac{R_S}{R_S + R_i} \right) \)
- \( R_i \ll R_i \)

- \( R_i = 0 \)

**If \( R_L = 0 \)**
\[ A_i = A_{iS} \approx \text{Short-circuit current gain} \]
Trans-conductance Amplifier:

\[ G_m = \text{ratio of short-circuit output current to input voltage} \]

\[ G_m \equiv \text{Short-circuit trans-conductance (A/V)} \]

Ideal Trans-conductance amplifier? \( R_i = \infty; R_o = \infty \)

Transresistance Amplifier:

\[ R_m = \text{ratio of open-circuit output voltage to input current} \]

\[ R_m \equiv \text{Open-circuit Transresistance} \]

Ideal Transresistance Amplifier?

\[ R_i = ? \]

\[ R_o = ? \]
Relationship between the 4 amplifier Models

Open-circuit output voltage?
For voltage amplifier: \( A_{N_0} \frac{V_i}{N_i} \)
For current amplifier: \( A_{i_s} i_c R_0 \)

\( A_{N_0} \frac{V_i}{N_i} = A_{i_s} i_c R_0 = A_{i_s} \left( \frac{V_i}{R_i} \right) R_0 \)

\[ A_{N_0} = A_{i_s} \frac{R_0}{R_i} \]

Similarly
\[ A_{N_0} = G_m R_0 \]
\[ A_{N_0} = \frac{R_m}{R_i} \]

These 3 equations can be used to relate any two of the gain parameters.

Measurement:

\( R_i \)?: Apply \( V_i \), measure \( i_c = \frac{V_i}{R_i} \)

\( R_o \)?:
1. Ratio of open-circuit output voltage to short-circuit output current
2. Eliminate input signals \((i_c \neq V_i = 0)\)

Apply test voltage \( V_x \) to output of amplifier

\[ R_o = \frac{V_x}{i_x} \]
Ex 1.10  Current amplifier \( \{ R_i = 100 \, \Omega \} \) is connected between a signal current source with \( 100 \, k\Omega \) source resistance and a \( 100 \, \Omega \) load. Voltage gain? Power gain?

\[
\begin{align*}
N_o &= 10^4 i_i \left( \frac{100 \, k\Omega}{100 \, \Omega} \right) \\

N_i &= R_i i_i \\

\Rightarrow N_o &= \frac{10^4 N_i}{100} \\

\text{Voltage gain} &= 10^4 \frac{V_o}{V_i} = 80 \, \text{dB}
\end{align*}
\]

\[
A_P = \frac{P_o}{P_i} = \frac{N_o^2}{N_i^2} \frac{100}{100} = 10^8 \frac{\text{W}}{\text{W}} \\
\Rightarrow 10 \log A_P = 80 \, \text{dB}
\]

Ex 1.13

Ri between B and Ground in the circuit above

\[
R_i = \frac{N_X}{i_X} \\
N_X = N_{BG} = \frac{r_h}{r_h + R_E (\beta + 1)} i_b
\]

\[
N_X = \left[ \frac{r_h + (\beta + 1) R_E}{r_h + (\beta + 1) R_E} \right] i_b = \left[ \frac{r_h + (\beta + 1) R_E}{r_h + (\beta + 1) R_E} \right] i_X
\]

\[
R_i = r_h + (\beta + 1) R_E
\]
Section 1.6

Linear Amplifier
Transfer Function

\[ V_{i} \sin \omega t = V_{0} \sin(\omega t + \phi) \]

\[ T(\omega) = \frac{|V_{0}|}{|V_{i}|} \]

\[ \angle T(\omega) = \phi \]

\[ T = |T|e^{i\phi} = \text{Re}T + j \text{Im}T \]

\[ e^{i\phi} = \cos\phi + j \sin\phi \]

\[ \text{Re}T = |T| \cos\phi \]

\[ \text{Im}T = |T| \sin\phi \]

\[ \phi = \tan^{-1}\left(\frac{\text{Im}T}{\text{Re}T}\right) \]

Example:

\[ R \quad \frac{1}{C} \quad N_{i} \quad \frac{N_{0}}{N_{i}} = \frac{1}{R + \frac{1}{jwC}} = \frac{1}{1 + jwRC} = T \]

\[ T = \frac{1 - jwRC}{\sqrt{1 + (wRC)^2}} = \text{Re}T + j \text{Im}T \]

\[ \phi = \tan^{-1}\left(-wRC\right) \]

\[ |T| = \frac{1}{\sqrt{1 + w^2RC^2}} \]
Low pass filter

Review

\( \frac{v(s)}{v_i(s)} = \frac{K}{1 + (\frac{s}{\omega_0})} \)

\( s = j\omega \)

\( T(j\omega) = \frac{1}{1 + i \left( \frac{\omega}{\omega_0} \right)} \)

\[ |T(j\omega)| = \frac{1}{\sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2}} \]

\[ 20 \log_{10} |T(j\omega)| = 20 \log_{10} \left( 1 + \left( \frac{\omega}{\omega_0} \right)^2 \right)^{-1/2} \]

- If \( \omega \to 0 \) (\( \omega \ll \omega_0 \)) \( \to 20 \log_{10} |T(j\omega)| \to 0 \) (dB)
- If \( \omega > \omega_0 \) \( \left( \frac{\omega}{\omega_0} \right)^2 \gg 1 \) \( \to 20 \log_{10} |T(j\omega)| = -20 \log_{10} \left( \frac{\omega}{\omega_0} \right) \)

**Bode Plot**

\[ -20 \log_{10} |T(j\omega)| \]

-20 dB

20 dB/decade

\( w_0 = \frac{1}{RC} \)

Corner or break frequency

Example

\( R \quad C \quad V_i \quad V_o \)

\( K = 1 \)

\( w_0 = \frac{1}{RC} = \omega_{3dB} \)
Figure 1.24  (a) Magnitude and (b) phase response of low-pass STC networks.
Figure 1.25  (a) Magnitude and (b) phase response of high-pass STC networks.

\[ 20 \log \left| \frac{T(j\omega)}{K} \right| \text{ (dB)} \]

\[ +20 \text{ dB/decade} \]

\[ -20 \text{ dB/decade} \]

\[ -30 \text{ dB/decade} \]

\[ 0 \]

\[ \frac{\omega}{\omega_0} \text{ (log scale)} \]

\( \phi(\omega) \)

\[ 90^\circ \]

\[ 5.7^\circ \]

\[ 45^\circ \]

\[ -45^\circ/\text{decade} \]

\[ 5.7^\circ \]

\[ 0 \]

\[ \frac{\omega}{\omega_0} \text{ (log scale)} \]

Fig. 1.25
<table>
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<th></th>
<th>Low-Pass (LP)</th>
<th>High-Pass (HP)</th>
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<tr>
<td><strong>Transfer Function</strong></td>
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<tr>
<td>$T(s)$</td>
<td>$\frac{K}{1 + (s/\omega_0)}$</td>
<td>$\frac{Ks}{s + \omega_0}$</td>
</tr>
<tr>
<td><strong>Transfer Function</strong></td>
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<td>for physical frequencies</td>
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<tr>
<td>$T(j \omega)$</td>
<td>$\frac{K}{1 + j(\omega/\omega_0)}$</td>
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<td><strong>Magnitude Response</strong></td>
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<tr>
<td>$</td>
<td>T(j \omega)</td>
<td>$</td>
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<tr>
<td><strong>Phase Response</strong></td>
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<tr>
<td>$\angle T(j \omega)$</td>
<td>$-\tan^{-1}(\omega/\omega_0)$</td>
<td>$\tan^{-1}(\omega_0/\omega)$</td>
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<td><strong>Transmission at</strong></td>
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<tr>
<td>$\omega = 0$ (dc)</td>
<td>$K$</td>
<td>0</td>
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<td><strong>Transmission at</strong></td>
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<tr>
<td>$\omega = \infty$</td>
<td>0</td>
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<td><strong>3-dB Frequency</strong></td>
<td>$\omega_0 = 1/\tau$, $\tau \equiv$ time constant $\tau = CR$ or $L/R$</td>
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<td><strong>Bode Plots</strong></td>
<td>in Fig. 1.24</td>
<td>in Fig. 1.25</td>
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</table>
Example 1.6

\[ \frac{V_o}{V_i} \text{?} \quad \text{DC gain?} \quad \text{3 dB frequency?} \]

1. **Input Side**
   \[ V_i = V_S \quad \frac{Z_i}{Z_i + R_S} = V_S \quad \frac{1}{1 + R_S \frac{1}{Z_i}} \]
   \[ Y_i = \frac{1}{Z_i} \]
   \[ V_i = V_S \cdot \frac{1}{1 + [1 + R_S] \frac{1}{R_i} + sC_i} \]
   We must write it in the form: \[ K/[1 + (\omega/\omega_0)] \]
   \[ \frac{V_i}{V_S} = \left[ 1 + \frac{R_S}{R_i} \right] \cdot \frac{1}{1 + \omega C_i \left[ \frac{R_S}{(R_S + R_i)} \right]} \]

2. **Output Side of Amplifier**
   \[ V_o = AV_i \quad \frac{R_L}{R_L + R_o} \]

   \[ \Rightarrow \text{Transfer Function} \]
   \[ \frac{V_o}{V_S} = A \left[ \frac{1}{1 + (R_S/R_i)} \right] \frac{1}{1 + (R_o/R_L)} \frac{1}{1 + \omega C_i \left[ \frac{R_S}{(R_S + R_i)} \right]} \]
   \[ \omega_0 = \frac{1}{C} \]

\[ \xi = C_i \left[ \frac{R_S}{(R_S + R_i)} \right] = C_i \left[ \frac{R_S}{R_i} \right] \]

**Quick way:**
\[ V_S = 0 \rightarrow C_i \text{ discharges through the parallel combination of } R_i \text{ and } R_S \]
\[ \xi = C_i \left[ \frac{R_S}{R_i} \right] \]
DC gain? 3dB frequency?

Frequency at which gain becomes 0 dB?

When $R_S = 20k \Omega$  
$C_i = 60pF$  
$R_0 = 200k \Omega$  
$R_1 = 100k \Omega$  
$M = 144$  
$R_L = 1k \Omega$.

$K = \frac{100V}{V} = 40 \text{ dB}$

$\omega_0 = 10^6 \text{ rad/s} \Rightarrow f_0 = \frac{\omega_0}{2\pi} = 159.2 \text{ kHz}$

Gain falls off at the rate of $-20 \text{ dB/decade}$ starting at $\omega_0$, the gain will fall at 0 dB in two decades.

\[ \Rightarrow \text{unit gain frequency} = 15.92 \text{ MHz} \]
It is required to couple a voltage source \( V_S \) having a resistance \( R_S \) to a load \( R_L \) via a capacitor \( C \). Derive an expression for the transfer function from source to load (i.e., \( \frac{V_L}{V_S} \)) and show that it is of the high-pass SFC type. For \( R_S = 10 \text{k}\Omega \) and \( R_L = 40 \text{k}\Omega \) find the smallest coupling capacitor that will result in a 3dB frequency no greater than 10 Hz.

\[
\frac{V_L}{V_S} = \frac{R_L}{R_S + \frac{1}{j\omega C} + R_L} = \frac{2R_L}{\omega (R_L + R_S) + \frac{1}{C}} = S + \frac{1}{C(R_S + R_L)}
\]

This transfer function is of the high-pass type with

\[
K = \frac{R_L}{R_S + R_L} \quad \text{and} \quad \omega_0 = \frac{1}{C(R_S + R_L)}
\]

For \( R_S = 10 \text{k}\Omega \) and \( R_L = 40 \text{k}\Omega \) to obtain a 3dB frequency \( \omega_0 < 2\pi \times 10 \text{ rad/s} \), we must select

\[
C \geq \frac{1}{\omega_0 (R_S + R_L)} = 0.318 \text{ nF}
\]
1.9 An amplifier with a voltage gain of +40 dB, an input resistance of 10 kΩ, and an output resistance of 1 kΩ is used to drive a load of 1 kΩ. 

What is the value of $A_{vo}$? Find the value of power gain $\Delta P$.

\[ A_{vo} = 40 \text{ dB} \]

\[ P_L = \frac{V_o^2}{R_L} = \frac{1}{R_L} \left( \frac{A_{vo} V_i}{R_L + R_o} \right)^2 \]

\[ P_L = 2.5 V_i^2 \]

\[ P_i = \frac{V_i^2}{R_i} = \frac{V_i^2}{10,000} \Rightarrow A_P = \frac{P_L}{P_i} = 2.5 \times 10^{-4} \text{ W/W} \]

\[ 10 \log A_P = 44 \text{ dB} \]

D1.12

\[ V_o = 17 - 10 V_i \] for \( 0.3 \text{ V} \leq V_o \leq 1 \text{ V} \)

Find $V_i$ so that the output voltage is 5 V.

If $V_i$ is superimposed on $V_i^*$, amplifier voltage gain is $\frac{V_o}{V_i}$.

If $V_i$ is sinusoidal, find its largest possible amplitude without output clipping.

\[ V_o (0.7, 10) \rightarrow \text{slope} = \text{gain} = -10 \text{ V/V} \]

\[ (5, 0.3) \]

To avoid clipping, the peak should be limited to $0.47 \text{ V}$.
15. An amplifier operating from a single 15 V supply provides a 12 V p-p sine-wave signal to a 1 kΩ load, and draws negligible input current from the signal source. The DC current drawn from the 15 V supply is 8 mA. What is the power dissipated in the amplifier and what is the power efficiency?

\[
P_{dc} = 15 \times 8 = 120 \text{ mW}
\]

\[
P_L = \frac{1}{2} V_{pp}^2 = \frac{1}{2} \left( \frac{6}{\sqrt{2}} \right)^2 = 102 \text{ mW}
\]

\[
\Rightarrow \eta = \frac{P_L}{P_{dc}} \times 100 = 15\%
\]

1.6. \( V_0 = 10 - 10^{-11} \ V \)

\[
V_I = V_0 + V_e
\]

\[
V_0 = V_0 + V_e \Rightarrow V_0 + V_e = 10 - 10^{-11} \ V
\]

\[
5 V
\]

\[
\Rightarrow V_0 = 5 - \left( 10^{-11} \times \frac{40 V_I}{5} \right) \times \frac{40 V_e}{5}
\]

\[
N_e = 2 \text{ mV} \rightarrow N_0 = -0.416 V
\]

\[
N_e = 5 \text{ mV} \rightarrow N_0 = -1.107 V
\]

\[
N_e = 10 \text{ mV} \rightarrow N_0 = -2.459 V
\]

For small signal gain \( N_0 = (-200 V/V) N_e \)