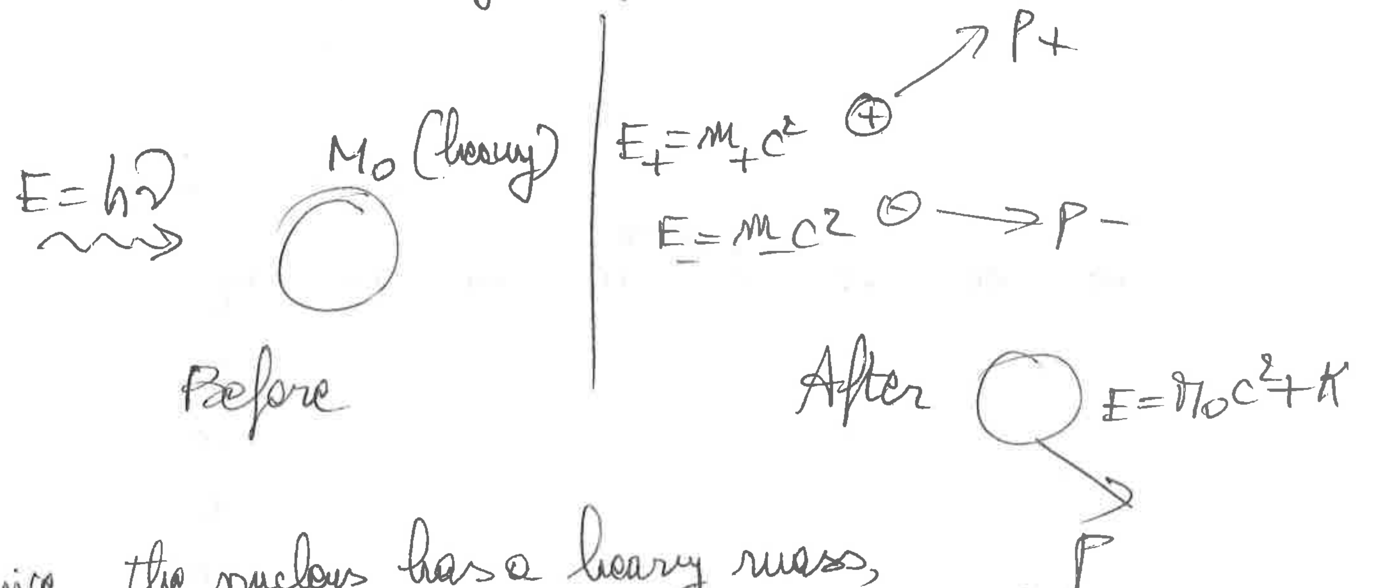
production and annihilation of pairs (1)

In a pair creation process, the photon energy is transformed into matter. A electron - positron pair is created. If the initial charge was zero, the positron must have a charge opposite to the electron.

For a pair to be created, the photon must have an energy at least equal to the total rest mass of the pair. The excess energy of the photon is transformed in kinetic energies of the particles created.



Since, the nucleus has a heavy mass, its kinetic energy after collision is negligible compared to E_+ and E_-, hence

$$h\nu = m_+ c^2 + m_- c^2 = K_+ + K_- + 2 M_0 c^2$$

positron & electron have the same rest mass m_0

Annihilation the e-positron pair disappears with the creation of 2 (or more) photons.
This can occur in vacuum.
For conservation of energy and momentum to occur, at least 2 photons must be created.

$$E_{initial} = E_{final} \qquad 2m_0c^2 + K_+ + K_- = h\nu_1 + h\nu_2$$

$$\vec{P}_{initial} = \vec{P}_{final} \qquad m_+\vec{v}_+ + m_-\vec{v}_- = \frac{h}{2\pi}\vec{k}_1 + \frac{h}{2\pi}\vec{k}_2$$

$$|\vec{k}| = \frac{2\pi}{\lambda}$$

A photon with $\lambda = 3{,}000\,\text{Å}$ produces an electron-positron pair near a nucleus. Find the kinetic energy of each particle if the energy of the positron is twice that of the electron.

Solution $\qquad E_{final} = E_{initial}$

$$\frac{hc}{\lambda} = 2m_0c^2 + K_+ + K_- = 2m_0c^2 + 3K_-$$

$$\frac{12.4\ 10^{-3}\ \text{MeV Å}}{0.0030\,\text{Å}} = 2(0.511\ \text{MeV}) + 3K_-$$

$$K_- = 1.04\ \text{MeV}$$

and $K_+ = 2K_- = 2.08\ \text{MeV}$

An electron of 5 MeV annihilates with a positron at rest and produces two photons. One of the photons moves in the direction of the incident electron. What is the energy of each photon created? (3)

The second photon must be moving parallel or antiparallel to the first photon.

So we must have

$$P_- = \frac{E_1}{c} + \varepsilon \frac{E_2}{c}$$ P_- is momentum of electron

with $\varepsilon = +1$ (parallel motion) / $\varepsilon = -1$ (antiparallel motion)

But $(K_- + m_0c^2)^2 = (P_-c)^2 + (m_0c^2)^2$

$\rightarrow P_-c = E_1 + \varepsilon E_2$

So $E_1 + \varepsilon E_2 = \sqrt{(K_- + m_0c^2)^2 - (m_0c^2)^2}$

$= \sqrt{(5.511\,\text{MeV})^2 - (0.511\,\text{MeV})^2}$

$E_1 + \varepsilon E_2 = 5.49\,\text{MeV}$ (1)

Conservation of energy gives

$E_1 + E_2 = K_- + m_0c^2 + m_0c^2$

$\boxed{E_1 + E_2 = 5\,\text{MeV} + 2\times(0.511\,\text{MeV}) = 6.02\,\text{MeV}}$ (2)

$\rightarrow E_1$ from (2) $\rightarrow$ (1)

$$-0.53\ MeV = (\varepsilon - 1)\,E_2$$

Only possible for $\varepsilon = -1$

$$\rightarrow E_2 = 0.27\ MeV$$

$$E_1 = 5.75\ MeV$$

Determine the maximum angle in a Compton scattering for which the photon can create an electron-positron pair.

To produce a pair, λ_s must be such that

$$h\frac{c}{\lambda_s} = 2m_0c^2$$

$$\text{or } 2\lambda_{s,\text{Threshold}} = \frac{h}{m_0 c}$$

Using Compton's result

$$\lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta) = 2\lambda_{s,Th}(1-\cos\theta)$$

$$\lambda' = \lambda + \underbrace{2\lambda_{s\,Th}(1-\cos\theta)}_{\geq \lambda_s}$$

$$\rightarrow \lambda' \geq \lambda_s$$

Equality $\quad 2\lambda_s(1-\cos\theta_s) = \lambda_s \Rightarrow \theta_s = 60°$

Absorption of photons

Combination of photoelectric, Compton effect, and pair production will lead to intensity attenuation for a beam incident on a material.

$$I = I_0 e^{-\alpha x}$$

α is absorption coefficient.

Example

Calculate the % of x-ray going through 5 mm of a material with absorption coefficient $\alpha = 0.07\ mm^{-1}$

$$\frac{I}{I_0} = e^{-\alpha x} = e^{-(0.07\ mm^{-1})(5.\ mm)} = 70.5\%$$

Derive the fact that $I = I_0 e^{-\alpha x}$

I reduction is due to photoelectric effect, Compton effect, and creation of e-h pairs.

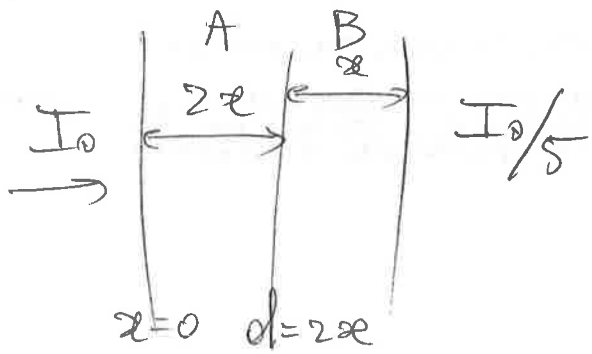
The number of reaction dN in space dx is directly proportional to the intensity or flux of photons and to the number of atoms which is proportional to dx

$$-dN = \alpha N\, dx$$

$$\rightarrow \int_{N_0}^{N} \frac{dN'}{N'} = -\alpha \int_0^x dx$$

$$N = N_0 e^{-\alpha x} \rightarrow I = I_0 e^{-\alpha x}$$

Two materials A & B have absorption coefficients $\alpha_A = 0.044 \text{ mm}^{-1}$ and $\alpha_B = 0.056 \text{ mm}^{-1}$.
If the incident energy is I_0 and the merging energy is $I_0/5$, what is the thickness of material A, B? A is twice as thick as B.



$$I(\text{on } B) = I_0 e^{-\alpha_A 2x}$$

$$I(3x) = I_0 e^{-2\alpha_A x} e^{-\alpha_B x}$$

$$I = I_0 e^{-(2\alpha_A + \alpha_B)x}$$

$$\rightarrow \quad 5 = e^{0.144 \text{ mm}^{-1} x}$$

$\rightarrow x = 11.18$ mm $\Rightarrow$ Thickness of B

$2x = 22.36$ mm $\equiv$ Thickness of A.