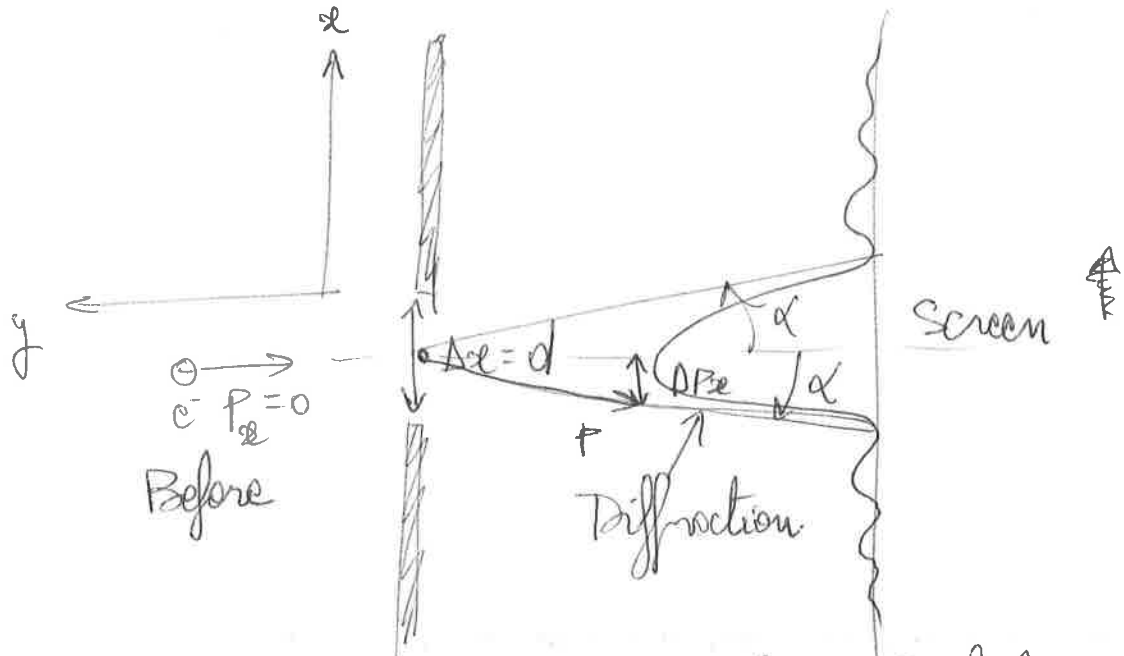
Heisenberg Uncertainty Principle



If particle hit screen, it went through hole. We know its location with accuracy $\Delta x = d$. We do not know exactly where the particle hit the screen. P_x is no longer zero after going through the hole. Since we do not know where the particle will hit the screen, there is an uncertainty on P_x, ΔP_x

What is the connection between Δx and ΔP_x?
If monochromatic wave with wavelength λ goes through a slit of width d, the wave will suffer some diffraction.
On the screen, the location of the first minimum is such that

$$\sin\alpha = \frac{\lambda}{d}$$

But De Broglie's relation gives

$$\lambda = \frac{h}{p}$$

The most likely region for the impact of the particle on the screen will be in the vicinity of the first maximum. So p_x will take value between 0 and $p \sin\alpha$

$$\Delta p_x = p \sin\alpha = p \frac{\lambda}{d} = \frac{h}{\lambda} \frac{\lambda}{d} = \frac{h}{d}$$

Δp_x can be made as small as possible by increasing d.

But $d = \Delta x$, so $\boxed{\Delta p \, \Delta x = h}$

Example Energy-time uncertainty relationship

A particle is moving along a straight line with kinetic energy $E = \frac{1}{2} m v^2$

Show that $\Delta E \Delta t \geq \frac{h}{4\pi}$ where $\Delta t = \frac{\Delta x}{v}$

$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m}$$

$$\Delta E = \frac{p \Delta p}{m} = \frac{mv}{m} \Delta p = v \Delta p$$

Bohr $\Delta p \Delta x \geq \frac{h}{4\pi}$

$$\longrightarrow \Delta E \geq \frac{h v}{4\pi \Delta x} = \frac{h}{4\pi} \frac{1}{\Delta t}$$

$$\boxed{\Delta E \Delta t \geq \frac{h}{4\pi}}$$

with $\Delta t = \frac{\Delta x}{v}$

What is the minimum uncertainty ~~on an~~ in energy for electronic state in which an electron stays for 10^{-8}s.

The time we have to measure the energy is 10^{-8}s.

$$\Delta E \, \Delta t \geq \frac{\hbar}{2}$$

$$\text{So } \Delta E \geq \underbrace{\frac{h}{4\pi \Delta t}}_{\Gamma = \frac{h}{4\pi \tau}} = \frac{hc}{4\pi c \Delta t} = 0.329 \; 10^{-7} \text{ eV}$$

$\hat{\tau}$ = mean lifetime of excited state - natural width of energy state

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The wavelength of a spectral ray has been measured to be 4,000 Å with uncertainty 10^{-4} Å.
What is the mean lifetime of the energy level which lead to this emission.

$$\Delta\lambda = 10^{-4} \text{ Å}$$

$$\hat{\tau} = \frac{h}{4\pi\Gamma} = \frac{h}{4\pi \Delta E}$$

for photon $E = \dfrac{hc}{\lambda} \rightarrow |\Delta E| = \dfrac{hc}{\lambda^2}(\Delta\lambda)$

$$\hat{\tau} = \frac{h}{4\pi\left(\frac{hc}{\lambda^2}\Delta\lambda\right)} = \frac{\lambda^2}{4\pi c \Delta\lambda} = \frac{(4 \, 10^{-7} m)^2}{4\pi (3 \, 10^{8} \frac{m}{s})(10^{-14} m)}$$

$$\boxed{\hat{\tau} = 4{,}24 \; 10^{-9} s}$$