Heisenberg Uncertainty Principle

If particle hit screen, it went through hole. We know its location with accuracy \( \Delta x = \lambda \). We do not know exactly where the particle hits the screen. \( \Delta x \) is no longer zero after going through the hole. Since we do not know where the particle will hit the screen, there is an uncertainty on \( \Delta x \), \( \Delta p_x \)

What is the connection between \( \Delta x \) and \( \Delta p_x \)?

If monochromatic wave with wavelength \( \lambda \) goes through a slit of width \( d \), the wave will suffer some diffraction. On the screen, the location of the first minimum is such that

\[
\sin \alpha = \frac{\lambda}{d}
\]
But de Broglie's relation gives

\[ \lambda = \frac{h}{p} \]

The most likely region for the impact of the particle on the screen will be in the vicinity of the first maximum. So \( p_x \) will take value between 0 and \( p \sin \alpha \).

\[ \delta p \sin \alpha = p \frac{d}{dt} \alpha = p \frac{d}{dt} \frac{A}{\lambda} = \frac{h}{\lambda} \frac{dA}{dt} = \frac{h}{p} \]

\( \delta p \) can be made as small as possible by increasing \( d \).

But \( d = \Delta x \), so \( \delta p \Delta x = h \)
Example: Energy-time uncertainty relationship

A particle is moving along a straight line with kinetic energy \( E = \frac{1}{2} m v^2 \).

Show that \( \Delta E \Delta t > \frac{\hbar}{4\pi} \) where \( \Delta t = \frac{\Delta x}{v} \).

\[ E = \frac{1}{2} m v^2 = \frac{p^2}{2m} \]
\[ \Delta E = \frac{p \Delta p}{m} = \frac{m v \Delta p}{m} = v \Delta p \]

\[ \Delta E \Delta p \Delta x > \frac{\hbar}{4\pi} \]

\[ \Rightarrow \Delta E > \frac{\hbar v}{4\pi \Delta x} = \frac{\hbar}{4\pi} \frac{1}{\Delta t} \]

\[ \boxed{\Delta E \Delta t > \frac{\hbar}{4\pi}} \]

With \( \Delta t = \frac{\Delta x}{v} \).
What is the minimum uncertainty in the energy of an electronic state in which an electron stays for $10^{-8}$ s. The time we have to measure the energy is $10^{-8}$.

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

$$\text{So } \Delta E \geq \frac{\hbar}{2 \gamma l \Delta t} = \frac{hc}{4 \pi l c \Delta t} = 0.329 \times 10^{-4} \text{ eV}$$

$$\gamma = \frac{\hbar}{4 \pi l c}$$

$\gamma$ = mean lifetime of excited state = natural width of energy state.

The wavelength of a spectral ray has been measured to be $4.1 \times 10^{-7}$ m with uncertainty $10^{-7}$. What is the mean lifetime of the energy level which lead to this emission?

$$\Delta \lambda = 10^{-7} \text{ m}$$

$$\Delta = \frac{\hbar}{4 \pi l c} = \frac{h}{4 \pi l c \Delta E}$$

for photon

$$E = \frac{hc}{\lambda} \Rightarrow (\Delta E) = \frac{hc}{\lambda^2} (\Delta \lambda)$$

$$\Delta = \frac{h}{4 \pi l c} (\frac{hc}{\Delta \lambda}) = \frac{\Delta \lambda^2}{4 \pi l c} = \frac{(4.1 \times 10^{-7})^2}{4 \pi l (3 \times 10^8 \text{ m/s}) (10^{-7} \text{ m})}$$

$$\Delta = 4.24 \times 10^{-9} \text{ s}$$