University of Cincinnati

Tunnel Diodes

30 GHz ⟷ 1cm
300 GHz ⟷ 1mm

To:

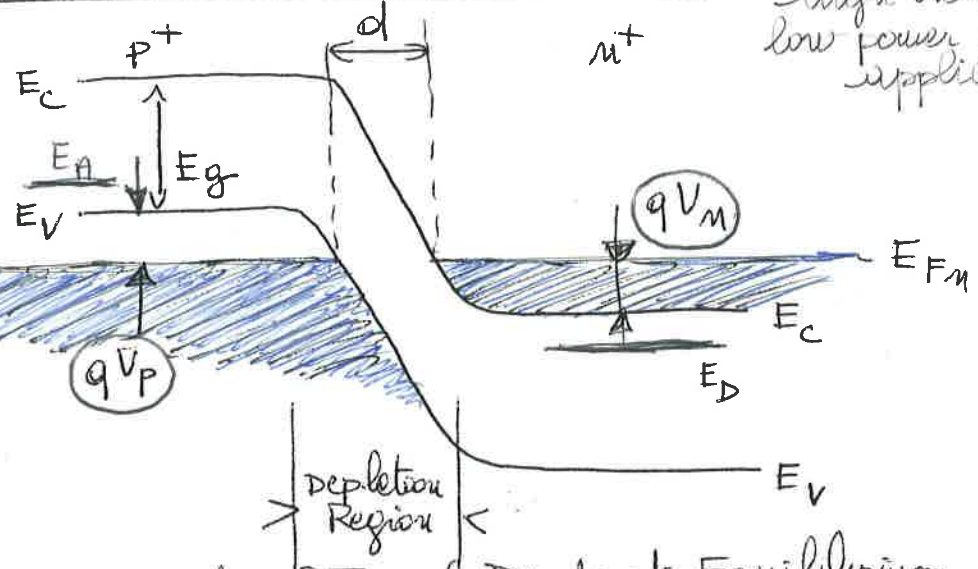
Date:

From: Big Sze pp 516 → 536

Tunnel Diode - Current Voltage Characteristics

(mature technology, high reliability, low power microwave applications)

At Thermal Equilibrium



Energy band Diagram of a Tunnel Diode at Equilibrium

Because of high doping, the depletion region is very narrow & the tunneling distance d is quite small (50 – 100 Å). The dopings also cause the Fermi levels to be located within the allowed bands.

$$V_p = (E_V - E_{Fp})/q \quad \text{are typically (a few kT or) } 50 \text{ to } 100\,mV.$$

$$V_n = (E_{Fn} - E_C)/q$$

Effects of High Doping:

One necessary condition for obtaining a tunnel diode is that both p-type & n-type semiconductor materials must be degenerate. We can calculate the doping density necessary to make the Fermi level just coincide with the edge of the band. Let us consider an n-type semiconductor in which we assume that the donor level E_D is a discrete level

$$n = N_C \frac{2}{\sqrt{\pi}} F_{1/2}\left(\frac{E_F - E_C}{kT}\right)$$

$$N_D^+ = N_D\left[1 - \frac{1}{1 + \frac{1}{g_D} \exp\left(\frac{E_D - E_F}{kT}\right)}\right] \qquad \rightarrow f(E_D) \leftarrow \text{probability for Donor level to be occupied.}$$

($g_D = 2$)

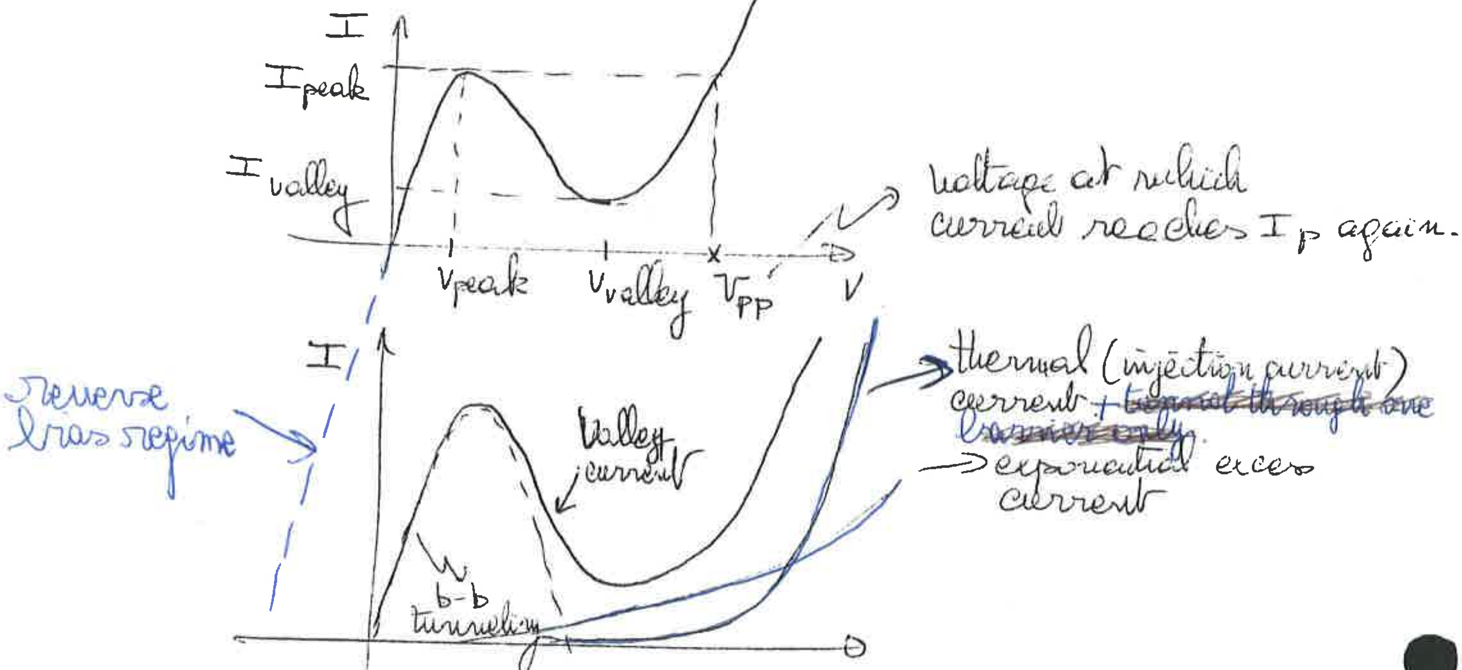
$$\left.\begin{array}{l} n \simeq N_D^+ \\ E_F - E_C = 0 \\ E_F - E_D = E_d \end{array}\right\} \Rightarrow N_D = N_C \frac{2}{\sqrt{\pi}} F_{1/2}(0)\left[1 + 2\exp\frac{E_d}{kT}\right] \simeq 0.68 N_C\left(1 + 2e^{\frac{E_d}{kT}}\right)$$

E_d = ionization energy

cc calculated from this last equation are about $2\,10^{19}\,cm^{-3}$ for Ge and $6\,10^{19}\,cm^{-3}$ for Si. for $E_D \approx 50\,meV$.

Tunneling Process

(1) Qualitative Consideration.



when a biasing voltage is applied, the electrons may tunnel from the valence band to the conduction or vice versa. The necessary conditions for tunneling are

(1) there are occupied energy states on the side from which electrons tunnel
(2) " " unoccupied " " at the same energy levels as in (1) on the side to which electrons can tunnel
(3) the tunneling potential barrier height should be low and the barrier width should be small enough that there is a finite tunneling probability.
(4) Momentum must be conserved in the tunneling process.

if forward voltage is applied such that the band is "uncrossed" i.e, the edge of the conduction band is exactly opposite to the top of the valence band, there are no available states opposite filled states this point, the tunneling current can no longer flow.
with further increase of voltage, the normal thermal current will flow and will increase exponentially with the applied voltage.

10b

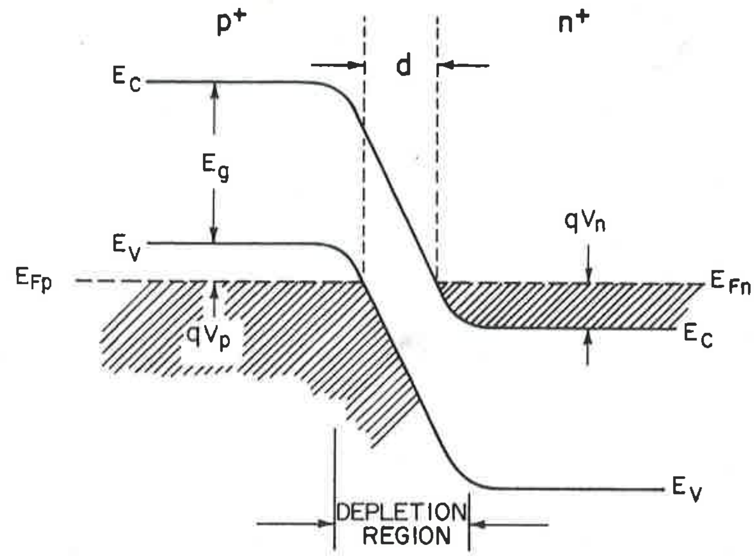


Fig. 2 Energy band diagram of a tunnel diode in thermal equilibrium.[1]

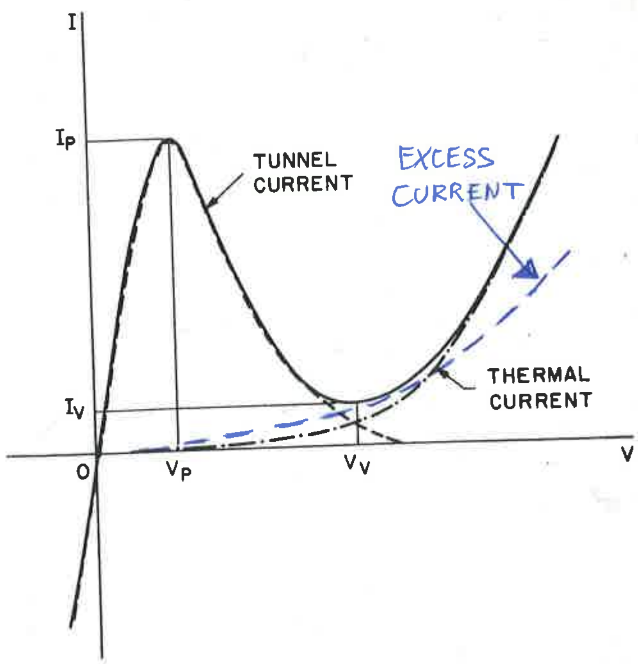


Fig. 3 Static current–voltage characteristics of a typical tunnel diode.

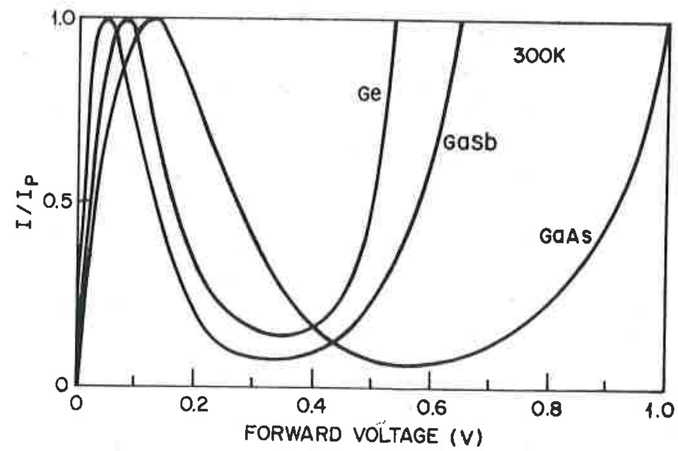


Fig. 5 Typical current–voltage characteristics of Ge, GaSb, and GaAs tunnel diodes room temperature.

E_g (Ge) = 0.66eV
E_g (GaSb) = 0.72eV
E_g (GaAs) = 1.42eV

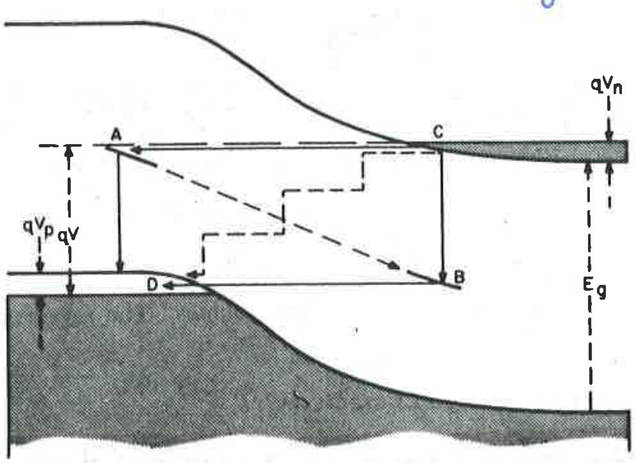


Fig. 9 Band diagram illustrating mechanisms of tunneling via states in the forbidden gap for the excess current flow. (After Chynoweth, Feldmann, and Logan, Ref. 13.)

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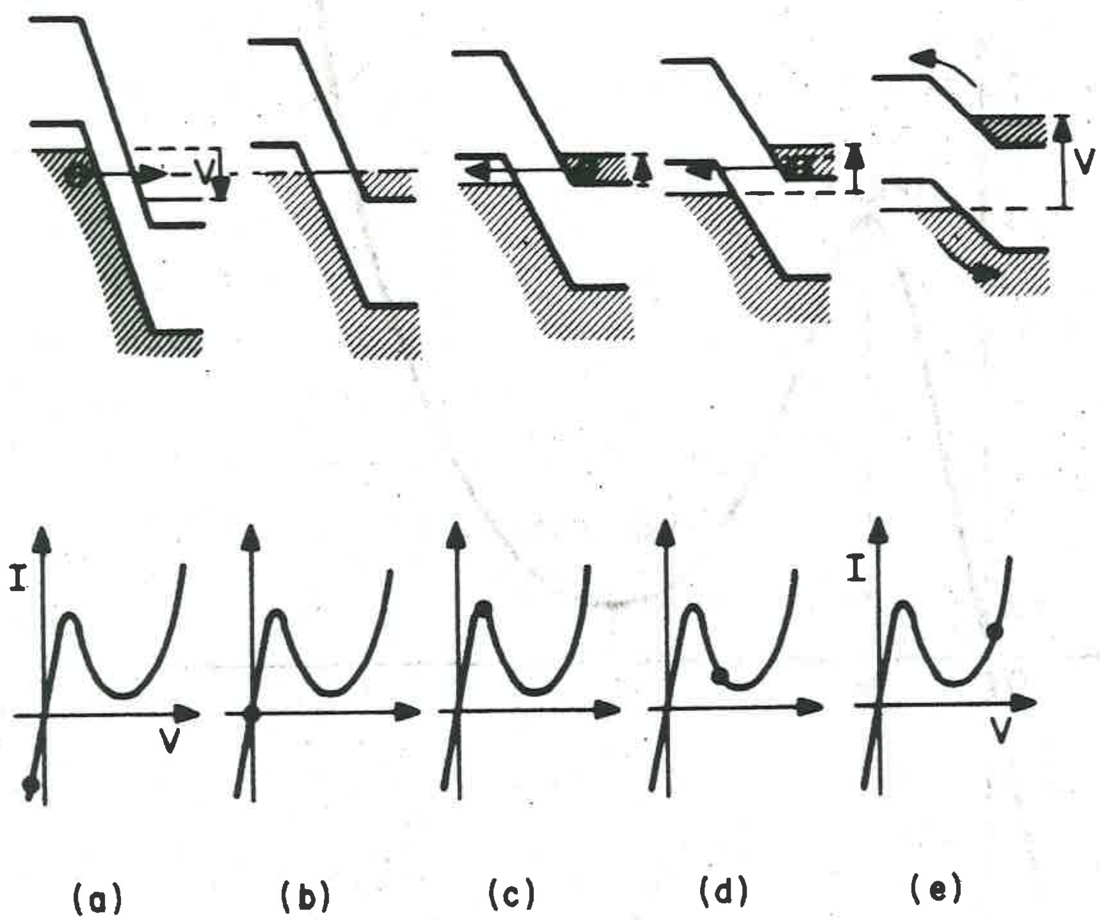


Simplified energy-band diagrams of tunnel diode at (a) reverse bias; (b) thermal equilibrium, zero bias; (c) forward bias such that peak current is obtained; (d) forward bias such that valley current is approached; and (e) forward bias with thermal current flowing. (After Hall, Ref. 3.)

at $T = 300K$ 10b

$\frac{I_P}{I_V} \sim \frac{8}{1}$ for Ge

$\sim \frac{12}{1}$ for GaSb, GaAs

$\sim \frac{4}{1}$ in Si

$\sim \frac{2}{1}$ in InAs

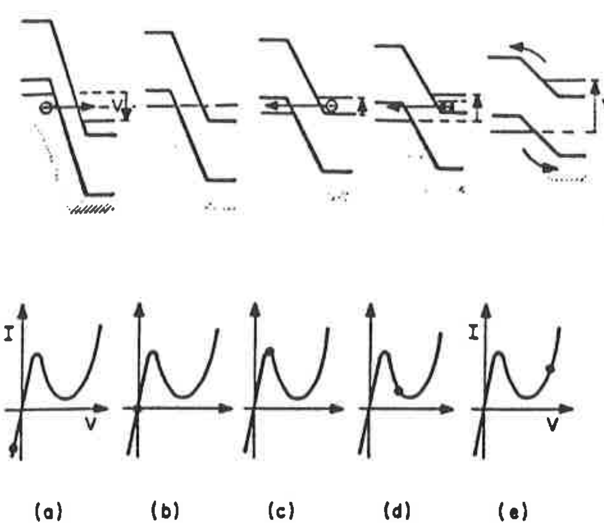
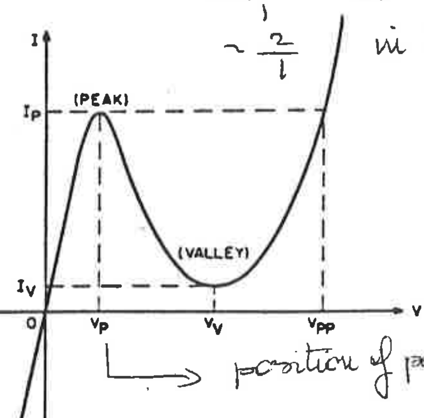


Fig 6 Simplified energy-band diagrams of tunnel diode at
(a) reverse bias,
(b) thermal equilibrium,
(c) forward bias such that peak current is obtained,
(d) forward bias such that the valley current is approached, and
(e) forward bias with thermal current flowing.
(After Hall, Ref. 48.)



position of peak ≈ $\frac{1}{3}(V_n + V_p)$ shifts towards higher values when doping ↑.

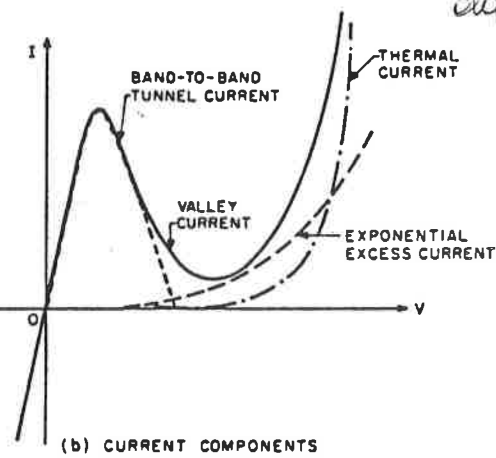
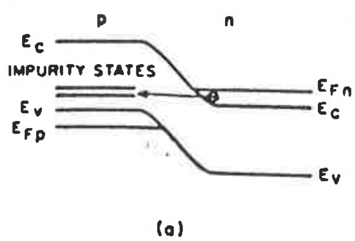
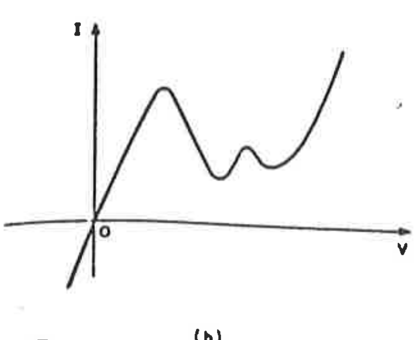


Fig. 5
(a) Static current-voltage characteristic of a typical tunnel diode. I_P and V_P are the peak current and peak voltage respectively. I_V and V_V are the valley current and valley voltage respectively. V_{PP} is the voltage at which the current again reaches I_P.
(b) The static current-voltage characteristic is decomposed into the current components.

Tunnel Diode and Backward Diode





band impurity conduction from A → B

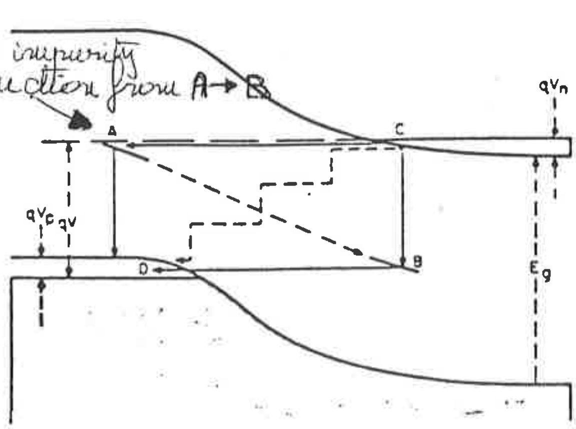


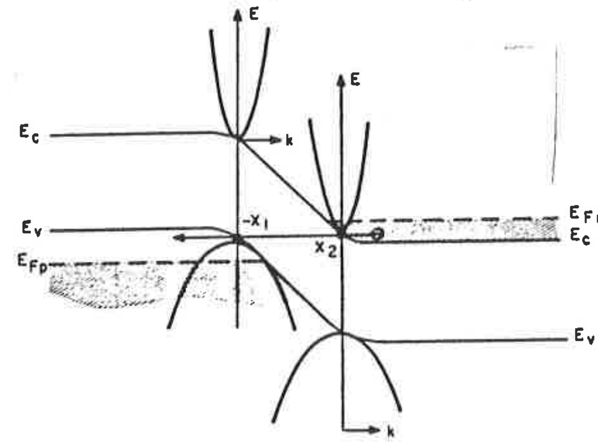
Fig. 14 Band diagram illustrating proposed mechanisms of tunneling via states in the forbidden gap for the excess current flow.
(After Chynoweth et al., Ref. 14.)

Phys. Rev., 121, p. 684 (1961)

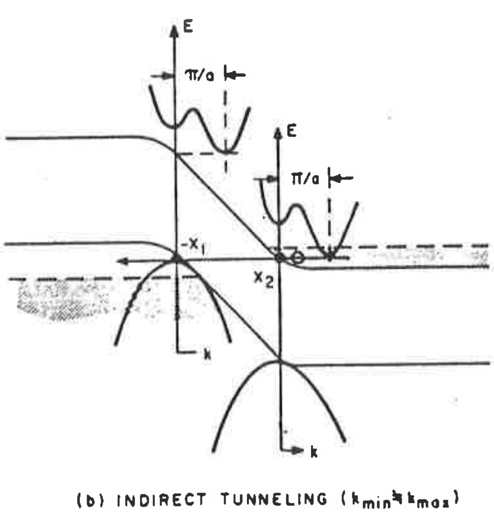
Fig. 16
(a) Tunneling into the narrow band of energies in the band gap.
(b) Current-voltage characteristic which has a hump current or a second negative resistance region.

One thus expects that as the forward voltage increases, the tunneling current increases from zero to a maximum I_p and then decreases to zero when $V = V_n + V_p$. The tunneling process can be either direct or indirect.

$\boxed{\text{Fig 8(a)}}$ For direct tunneling to occur, the E_c min and the valence band max must have the same momentum. This condition can be fulfilled by semiconductors $\boxed{\text{such as GaAs \& GaSb}}$ that have a direct bandgap. For indirect tunneling, the E_c min. does not occur at the same momentum as the valence band max. In order to conserve momentum, the difference in momentum between the E_c min and E_V max must be supplied by scattering agents such as phonons or impurities. In general, the probability for indirect tunneling is much lower than the probability for direct tunneling when direct tunneling is possible. Also indirect tunneling involving several phonons has a much lower probability than that with a single phonon.



(a) DIRECT TUNNELING ($k_{min} = k_{max}$)



(b) INDIRECT TUNNELING ($k_{min} \neq k_{max}$)

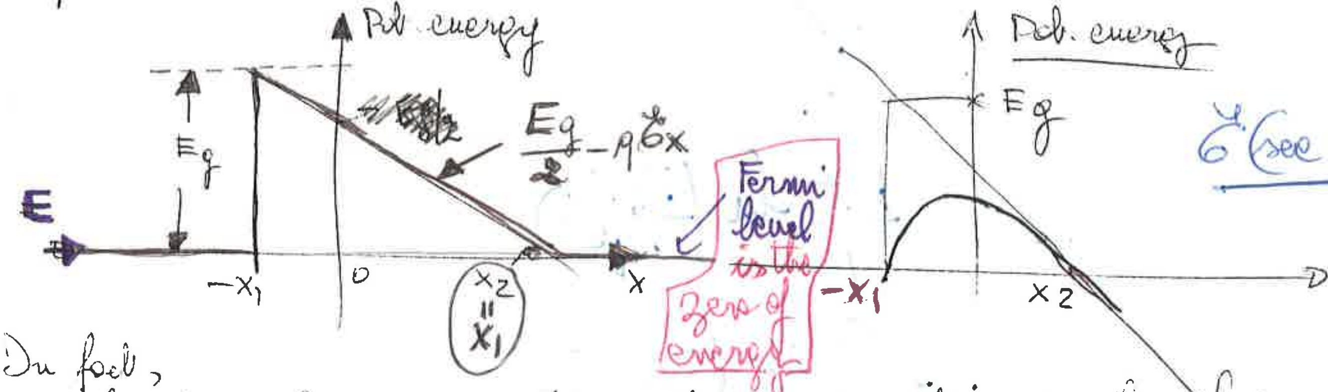
Fig. 8
(a) Direct tunneling process with *E-k* relationship at the classical turning points ($-x_1$ and x_2) superimposed on the *E-x* relationship of the tunnel junction.
(b) Indirect tunneling process where $k_{min} \neq k_{max}$.

Tunneling probability & Tunneling Current

When the $\mathcal{E}$ field in a semiconductor is sufficiently high ($\simeq 10^6$ V/cm), there is a finite probability of quantum tunneling or direct excitation of electrons from the valence band into the conduction band.

WKB approximation.
$$T_t \simeq \exp\left[-2\int_{-x_1}^{x_2} |k(x)|\,dx\right]$$

where $|k(x)|$ is the absolute value of the wave vector of the carrier in the barrier. $-x_1$ & x_2 are the classical turning points shown in the previous figure.
The tunneling of an electron through a forbidden band is formally the same as a particle tunneling through a barrier. At the present time the detailed form of the potential barrier for an electron in the forbidden gap is not known.
people have used the triangular & Parabolic potential barriers.



$\mathcal{E}$ (see back)

In fact,
The tunneling exponent is not very sensitive to the choice of the potential barriers.
$\mathcal{E} = \mathcal{E}$ at metallurgic junction (from diode theory: see Streetman's book for instance)

For Δ barrier,
$$k(x) = \sqrt{\frac{2m^*}{\hbar^2}(PE - E)} = \sqrt{\frac{2m^*}{\hbar^2}\left(\frac{E_g}{2} - q\mathcal{E}x\right)}$$
$$\rightarrow T_t \simeq \exp\left[-2\int_{-x_1}^{x_2}\sqrt{\frac{2m^*}{\hbar^2}\left(\frac{E_g}{2} - q\mathcal{E}x\right)}\,dx\right]$$
$$T_t \simeq \exp\left[\frac{4}{3}\frac{\sqrt{2m^*}}{q\mathcal{E}\hbar}\left(\frac{E_g}{2} - q\mathcal{E}x\right)^{3/2}\right]_{-x_1}^{x_2}$$
$$x = x_2\,;\ \frac{E_g}{2} - q\mathcal{E}x = 0 \quad \Big\| \quad x = -x_1 \quad \left(\frac{E_g}{2} - q\mathcal{E}x\right) = E_g$$
we therefore have
$$T_t \simeq \exp\left(-\frac{4\sqrt{2m^*}\,E_g^{3/2}}{3q\hbar\mathcal{E}}\right)$$

Current?

At thermal $\rightleftharpoons$, the tunneling current $I_{V \to C}$ from the valence band to to the empty state of E_C & the current $I_{C \to V}$ from $E_C \to$ empty state of the valence band should be detail-balanced

$$I_{C \to V} = A \int_{E_C}^{E_V} F_C(E)\, g_C(E)\, T_t \{1 - F_V(E)\}\, g_V(E)\, dE$$

$$I_{V \to C} = A \int_{E_C}^{E_V} F_V(E)\, g_V(E)\, T_t \{1 - F_C(E)\}\, g_C(E)\, dE$$

where A is a constant and T_t is assumed to be the same for both directions. $F_C(E)$, $F_V(E)$ are the F-D distribution functions.

$n_C(E)$, $n_V(E)$ are the density of states in the conduction band & valence bands respectively.

$$\boxed{g_C(E) = M_C \frac{\sqrt{2}}{\pi^2} \frac{\sqrt{E - E_C}}{\hbar^3} \left(m_{de}\right)^{3/2}} \qquad \boxed{F(E) = \frac{1}{1 + e^{\frac{E - E_F}{kT}}}}$$

When the junction is biased

$$I = I_{C \to V} - I_{V \to C} = A \int_{E_C}^{E_V} [F_C(E) - F_V(E)]\, T_t\, g_C(E)\, g_V(E)\, dE$$

A closed form of the above expression can be obtained under the following assumptions: (1) Tunneling T_t is almost a constant in the small voltage range involved, (2) The densities of states in E_C and E_V vary as $(E - E_C)^{1/2}$ and $(E_V - E)^{1/2}$ respectively and (3) both V_n & V_p are equal or less than $2kT$. With the above assumptions, $F_{C,V}(E)$ can be approximated by linear functions of E, i.e,

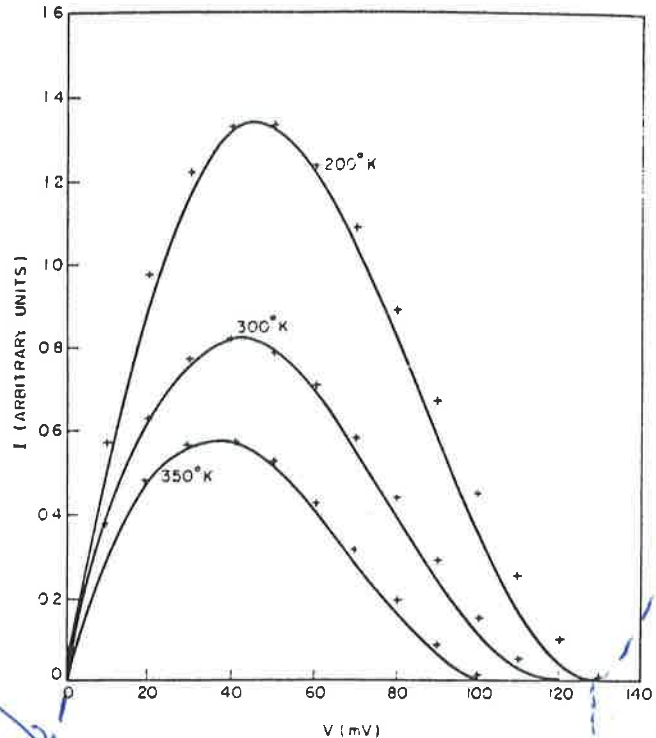
$$F_C(E) \simeq \frac{1}{2} - (E - E_{Fn})/4kT$$
$$F_V(E) \simeq \frac{1}{2} + (E_{Fp} - E)/4kT$$

$$\longrightarrow \boxed{I = \underbrace{A'}_{\text{constant}} T_t \left(\frac{qV}{kT}\right) (V_n + V_p - V)^2} \Rightarrow \text{gives reasonable agreement especially at room \& elevated Temperature.}$$

V is the applied voltage

The next figure shows a comparison between theoretical and experimental data. The constant A' is chosen to fit the peak current values. There is a reasonable agreement at room temperature and above.

(*) using the expression for I above, show that $\left.\frac{\partial I}{\partial V}\right) = 0$ when $qV = \frac{1}{3}(V_n + V_p) \equiv$ position of peak current



unphysical

cubic

unphysical

$V_n + V_p$ ← function of temperature

Fig. 10 Comparison of experimental curves and the theoretical results. (After Karlovsky, Ref. 11.)

Excess Current : See figure 14

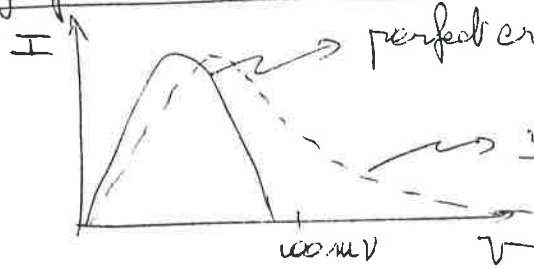
For ideal diode, the current should decrease to zero for $V \geq V_n + V_p$.

2 Major components of excess current

(1) valley current due to band tail tunneling.

(2) Exponential excess current due to carrier tunneling by way of energy states within the forbidden gap. Other possible mechanisms leading to excess current such as those due to photon, phonon, or plasmon have been considered and found to be too small to be important.

Effect of density of states tails on I-V characteristic of tunnel junction



Exponential Excess current (Chynoweth et. al.) Phys. Rev. Vol. 121, p. 684 (1961)

4 different routes (see fig. 14)

$I_{excess} \propto \exp(\alpha T)\exp(\beta V)$

By contrast injection current varies as $\exp(qV/kT)$

Also, $I_{excess} \propto \exp(\beta V)$ which has also been confirmed experimentally.

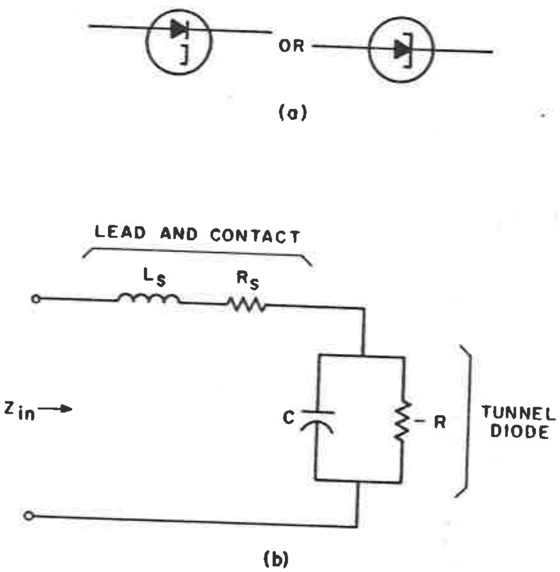


Fig. 12 (a) Symbol of tunnel diode. (b) Equivalent circuit of tunnel diode. (After Ref. 28.)

The series resistance R_S includes the lead resistance, the ohmic contacts, and the spreading resistance in the wafer, which is given by $\rho/2d$, where ρ is the resistivity of the semiconductor and d is the diameter of the diode area. The series inductance L_S in a coaxial cavity is given by[29]

$$L_S = \frac{2.303\mu_0 l}{2\pi} \ln\left(\frac{r_2}{r_1}\right) \tag{33}$$

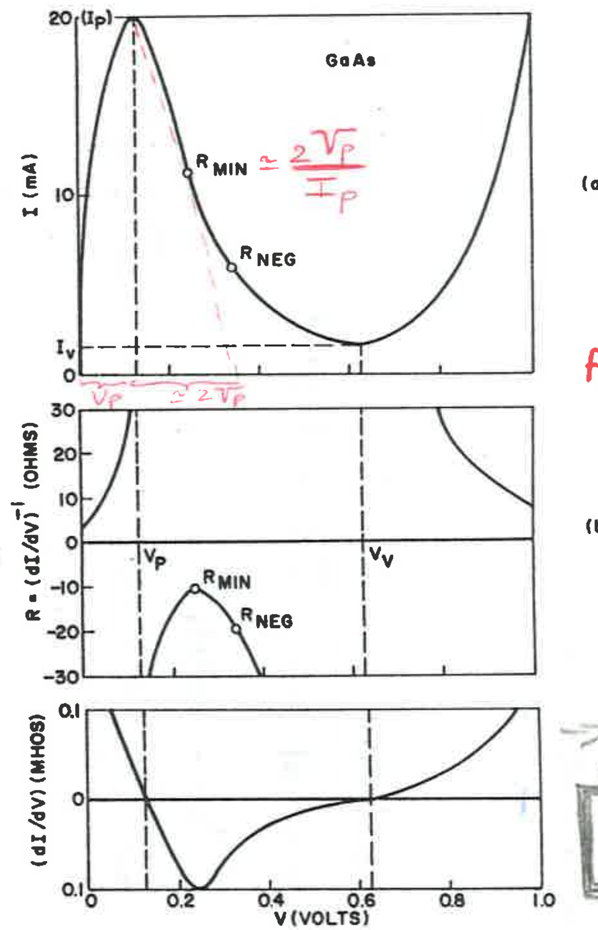
where μ_0 is the permeability of the medium, l is the length, and r_1 and r_2 are the inner and outer radii of the coaxial line, respectively. We shall see that these parasitic elements establish important limits on the performance of the tunnel diode.

To consider the diode capacitance and negative resistance, we refer to typical current–voltage characteristic, Fig. 13*a*. The differential resistance, defined as $(dI/dV)^{-1}$, is plotted in Fig. 13*b*. The value of the negative resistance at the inflection point, which is the minimum negative resistance in the region, is designated by R_{min}. This resistance can be approximated by

$$R_{min} \approx 2V_P/I_P \tag{34}$$

where V_P and I_P are the peak voltage and peak current, respectively. Figure 13*c* shows the conductance plot (dI/dV) versus V. At the peak and valley voltages the conductance becomes zero; the diode capacitance is usually measured at the valley voltage, and is designated by C_j.

Microwave 300 GHz ⟷ $\lambda = 1$ mm
30 GHz ⟷ $\lambda = 1$ cm



$R_{MIN} \simeq \frac{2V_P}{I_P}$

Here

$V_P \simeq 0.16$

$I_P = 20$ mA

$R_{min} \simeq \frac{0.32}{0.02}$

$\boxed{R_{min} \simeq 16\,\Omega}$

Fig. 13 (a) Current–voltage characteristics of a GaAs tunnel diode at 300 K. (b) Differential resistance $(dI/dV)^{-1}$ versus voltage, where R_{min} is the minimum resistance and R_{neg} is the resistance corresponding to the minimum noise figure. (c) Differential conductance $G \equiv dI/dV$ versus voltage. At peak and valley currents $G = 0$.

The input impedance Z_{in} of the equivalent circuit of Fig. 12 is given by

$$Z_{in} = \left[R_S + \frac{-R}{1+(\omega RC)^2}\right] + j\left[\omega L_S + \frac{-\omega CR^2}{1+(\omega RC)^2}\right]. \tag{35}$$

From Eq. 35 we see that the resistive (real) part of the impedance will be zero at a certain frequency, and the reactive (imaginary) part of the impedance will also be at a second frequency. We denote these frequencies by the resistive cutoff frequency f_r and the reactive cutoff frequency f_x,

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(15)

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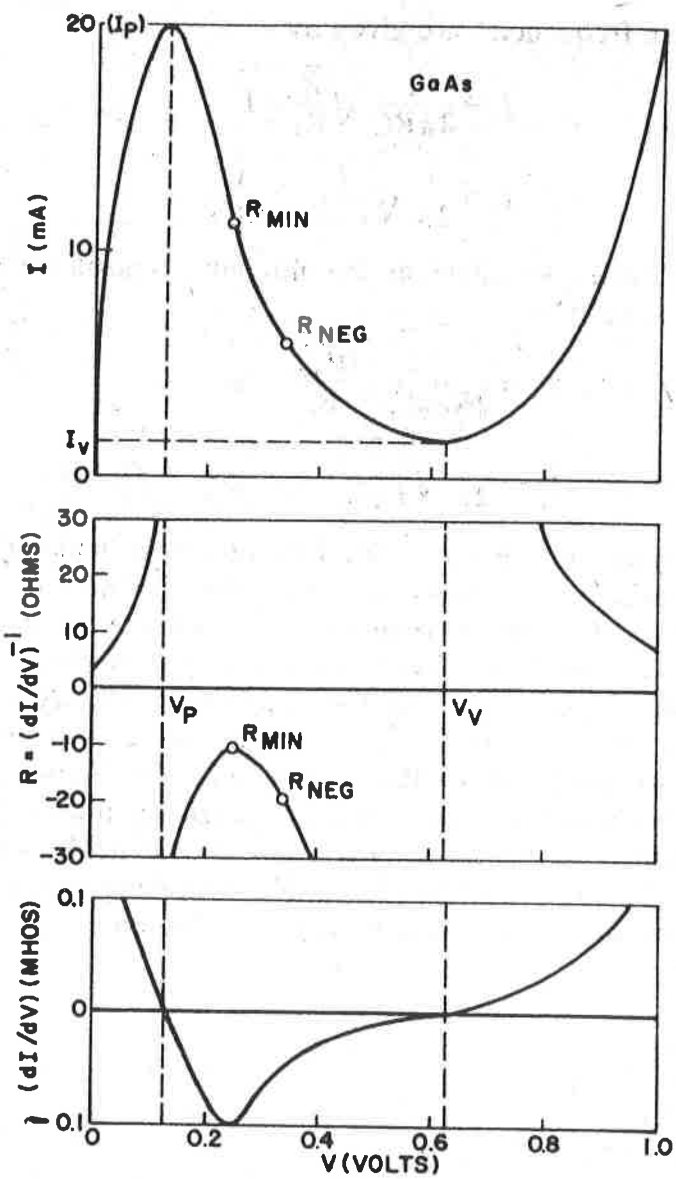
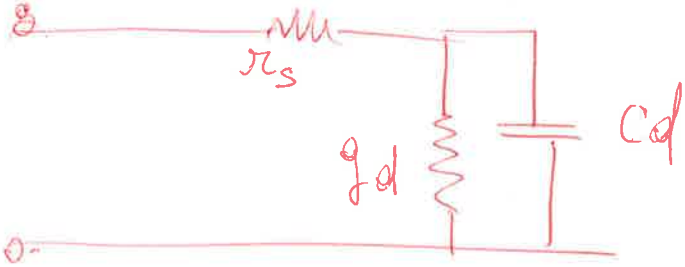


Fig. 13 (*a*) Current–voltage characteristics of a GaAs tunnel diode at 300 K. (*b*) Differential resistance $(dI/dV)^{-1}$ versus voltage, where R_{min} is the minimum resistance and R_{neg} is the resistance corresponding to the minimum noise figure. (*c*) Differential conductance $G \equiv dI/dV$ versus voltage. At peak and valley currents $G = 0$.

Additional Notes



r_s ⇒ bulk resistance of material + additional lead contact resistance

At low frequencies, $r_s + r_d \triangleq$ slope of I-V curve.

$C_d \triangleq$ Depletion layer capacitance of p-n junction.

For a given Impurity cc., I_p is $\doteqdot$ to Junction Area
so is Cd

$\longrightarrow$ I_p / C_d

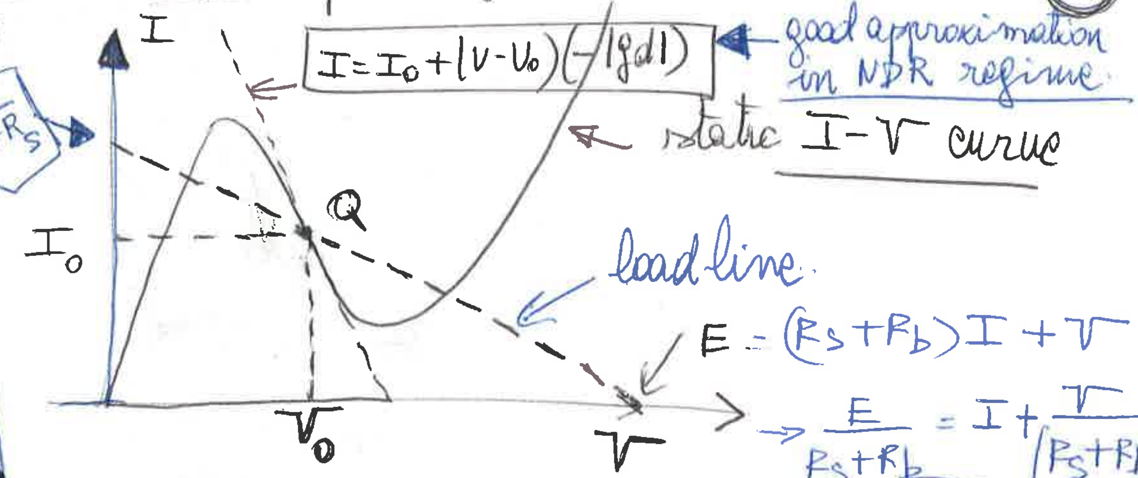
$$\frac{C_d}{A} = \sqrt{\frac{q\varepsilon}{2}} \sqrt{\frac{np}{p+n}} \left(V_d - V\right)^{-1/2}$$

- A = junction area
- q = electron charge
- ε = dielectric constant
- n, p = electron & hole cc.
- V_d = built-in potential (diffusion potential)
- V = applied voltage

$\boxed{\text{Cd varies with bias. Take value of Cd around } V = V_{valley}}$

Inherent Stability Diagram of Tunnel Diodes

Stability Criteria



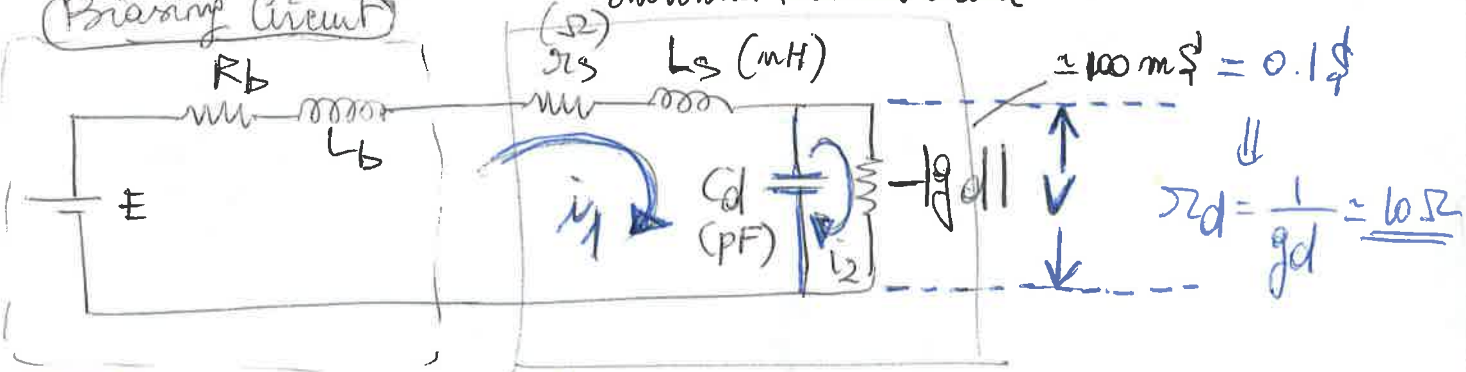
Source: Chow "Principles of Tunnel Diode Circuits" on TK 7872.D6C5 pp 76-81

$$\rightarrow \frac{E}{R_s + R_b} = I + \frac{V}{[R_s + R_b]}$$

$$I = \frac{V - E}{[R_s + R_b]}$$

$$\left[\frac{\partial I}{\partial V}\right]_Q = -|g_d|$$

To secure the dc bias point, while avoiding bistability we want $(r_b + r_s) < \frac{1}{|g_d|} = r_d$



if $R_b \neq 0$ $\quad (r_s + R_b)|-g_d| < 1$ is one stability criteria

For a small region around Q point (V_0, I_0)

$$I = I_0 + (V - V_0)(-|g_d|)$$

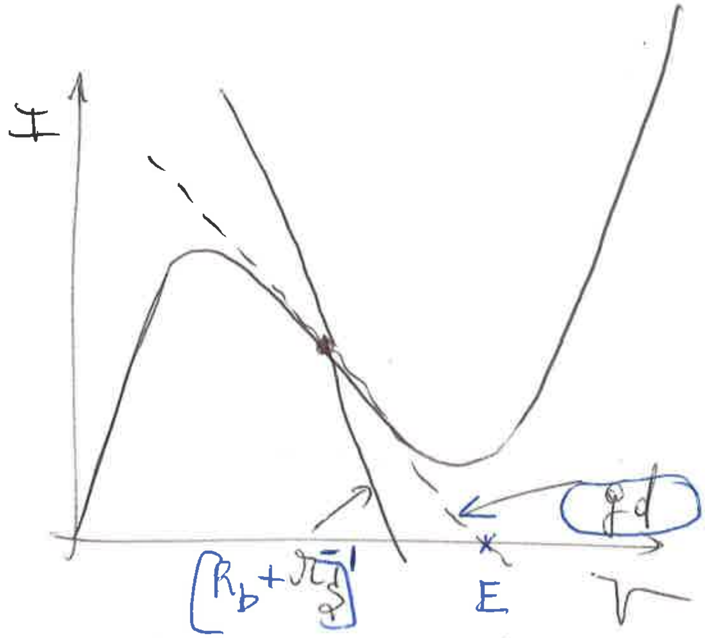
or

$$V = \frac{I_0 - I + V_0|g_d|}{|g_d|} \rightarrow V = \frac{I_0 - i_2 + V_0|g_d|}{|g_d|}$$

$$E = (R_b + r_s)i_1 + (L_b + L_s)\frac{di_1}{dt} + \frac{1}{C_d}\int i_1 dt - \frac{1}{C_d}\int i_2 dt$$

$$V = \frac{1}{C_d}\left(\int i_1 dt - \int i_2 dt\right) =$$

[Voltage across capacitor]



$R_b, L_b = 0$
Neglect C_d, L_s

Intersection at one point only in NDR if $[r_s + r_b]^{-1} > g_d$

or $\boxed{r_s g_d < 1}$ Stability criteria

If $r_s g_d > 1 \rightarrow$ 3 points of intersection $\rightarrow$ logic circuits.

$$\frac{-1}{(R_s + R_b)} < -g_D$$

or

$$\frac{1}{R_s + R_b} > g_D$$

$$\Rightarrow \boxed{(R_s + R_b) g_D < 1}$$ one point of intersection

Otherwise 3 points of intersection

$$\boxed{\begin{array}{l} L = L_b + L_s \\ R = R_b + r_s \end{array}}$$

Intermediate steps

$\boxed{V \text{ varies with } t \text{ because } i_2 \text{ does.}}$

$$E = L\frac{di_1}{dt} + Ri_1 + V$$

$$V = \frac{I_0 - i_2 + V_0|gd|}{|gd|} = V_0 + \left(\frac{I_0 - i_2}{|gd|}\right)$$

$$E = L\frac{di_1}{dt} + Ri_1 + \frac{I_0 + V_0|gd| - i_2}{|gd|}$$

$$\Rightarrow \quad L\frac{di_1}{dt} + Ri_1 - \frac{i_2}{|gd|} + \frac{I_0 + (V_0 - E)|gd|}{|gd|} = 0$$

∴ differentiating with respect to t the Eq. at the bottom of previous page

$$\Rightarrow \quad i_2 = C\left(L\frac{d^2 i_1}{dt^2} + R\frac{di_1}{dt} + \frac{i_1}{C}\right)$$

Substituting this i_2 into

we get the following second order differential equation for i_1

$$L\frac{d^2 i_1}{dt^2} + \left(R - \frac{L|gd|}{Cd}\right)\frac{di_1}{dt} + \left(\frac{1 - R|gd|}{Cd}\right)i_1 = \frac{I_0 + |gd|(V_0 - E)}{Cd}$$

General Solution

$$\Rightarrow \boxed{i_1 = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + \left\{\frac{I_0 + |gd|(V_0 - E)}{1 - R|gd|}\right\}}$$

also

$$\frac{I_o - i_2 + V_o|g_d|}{|g_d|} = \frac{1}{C_d}\left(\int i_1 dt - \int i_2 dt\right) = V$$

$$\Rightarrow L\frac{d i_1^2}{dt^2} + \left(R - \frac{L g_d}{C_d}\right)\frac{d i_1}{dt} + \left(\frac{1 - R g_d}{C_d}\right) i_1 = \frac{I_o + g_d(V_o - E)}{C_d}$$

with $\begin{cases} L = L_b + L_s \\ R = R_b + r_s \end{cases}$

General Solution

$$\Rightarrow \quad i_1 = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} + \left\{\frac{I_o + |g_d|(V_o - E)}{1 - R|g_d|}\right\}$$

$$\therefore \lambda_{1,2} = \frac{1}{2}\left(\frac{g_d}{C_d} - \frac{R}{L}\right) \pm \sqrt{\frac{1}{4}\left(\frac{R}{L} - \frac{g_d}{C_d}\right)^2 - \left(\frac{1 - R g_d}{L C_d}\right)}$$

If $Re(\lambda_1, \lambda_2) > 0 \longrightarrow$ circuit is unstable

If $L_b, R_b = 0 \longrightarrow$ Stability criteria of a tunnel diode

$$\lambda_{1,2} = \frac{1}{2}\left(\frac{g_d}{C_d} - \frac{r_s}{L_s}\right) \pm \sqrt{\frac{1}{4}\left(\frac{r_s}{L_s} - \frac{g_d}{C_d}\right)^2 - \left(\frac{1 - r_s g_d}{L_s C_d}\right)}$$

λ real $\longrightarrow$ initial disturbance will either grow or decay exponentially

λ complex $\longrightarrow$ transient will be growing or decaying sinusoids.

The equations for $\lambda_{1,2}$ involve 4 parameters

→ Normalized $\lambda_{1,2}$

Introduce 2 variables
$$\begin{cases} \omega_0 = \dfrac{1}{\sqrt{L_s C_d}} \\ Q_d = \dfrac{\omega_0 C_d}{g_d} \end{cases}$$

$\boxed{\omega_0 = 2\pi f}$

$\rightarrow f = \dfrac{1}{2\pi}\dfrac{1}{\sqrt{L_s C_d}}$

$L_s = 0.1\,mH$

$C_d = 4\,pF$

$\rightarrow f = \dfrac{4}{2\pi}\dfrac{1}{\sqrt{4\,10^{-12}\,10^{-10}}} = \dfrac{10^{11}}{4\pi} \simeq 2GHz$

$$\Rightarrow \frac{\lambda_{1,2}}{\omega_0} = \frac{1}{2}\left(\frac{1}{Q_d} - r_s g_d Q_d\right) \pm \sqrt{\frac{1}{4}\left(r_s g_d Q_d - \frac{1}{Q_d}\right)^2 - (1 - r_s g_d)}$$

y x y x y

If $\boxed{r_s g_d \rightarrow 0}$

$$\frac{\lambda_{1,2}}{\omega_0} = \frac{1}{2Q_d} \pm \sqrt{\frac{1}{4Q_d^2} - 1}$$

$\lambda_{1,2}$ real for $Q_d < 0.5$
complex for $Q_d > 0.5$

when $\boxed{r_s g_d = 1}$ ⟹ $\lambda_1 = 0$ & $\lambda_2 = \omega_0\left(\dfrac{1}{Q_d} - Q_d\right)$

$\lambda_2 > 0$ for $Q_d < 1$
and $\lambda_2 < 0$ for $Q_d > 1$

$\boxed{\text{for } r_s g_d > 1}$, $\lambda_{1,2}$ are always ~~> 0 &~~ real & > 0.
⟹ Transient to a bistable state.

In practice r_s is always smaller than $1/g_d$
→ bistable circuit you must add an external resistance ~~with~~ in series with r_s.

$y = R_s g_d \qquad x = Q_d$

$$\frac{\lambda_{1,2}}{\omega_0} = \frac{1}{2}\left(\frac{1}{x} - xy\right) \pm \sqrt{\frac{1}{4}\left(xy - \frac{1}{x}\right)^2 - (1-y)}$$

$$= \frac{1}{2}\left(\frac{1}{x} - xy\right) \pm \sqrt{\frac{1}{4}\left(xy + \frac{1}{x} + 2\right)\left(xy + \frac{1}{x} - 2\right)}$$

$x \geq 0$
$y \geq 0 \implies xy + \frac{1}{x} + 2 \geq 0.$

So we can determine if $\lambda_{1,2}$ are real or complex by looking at sign of $\boxed{xy + \frac{1}{x} - 2 = \Delta}$

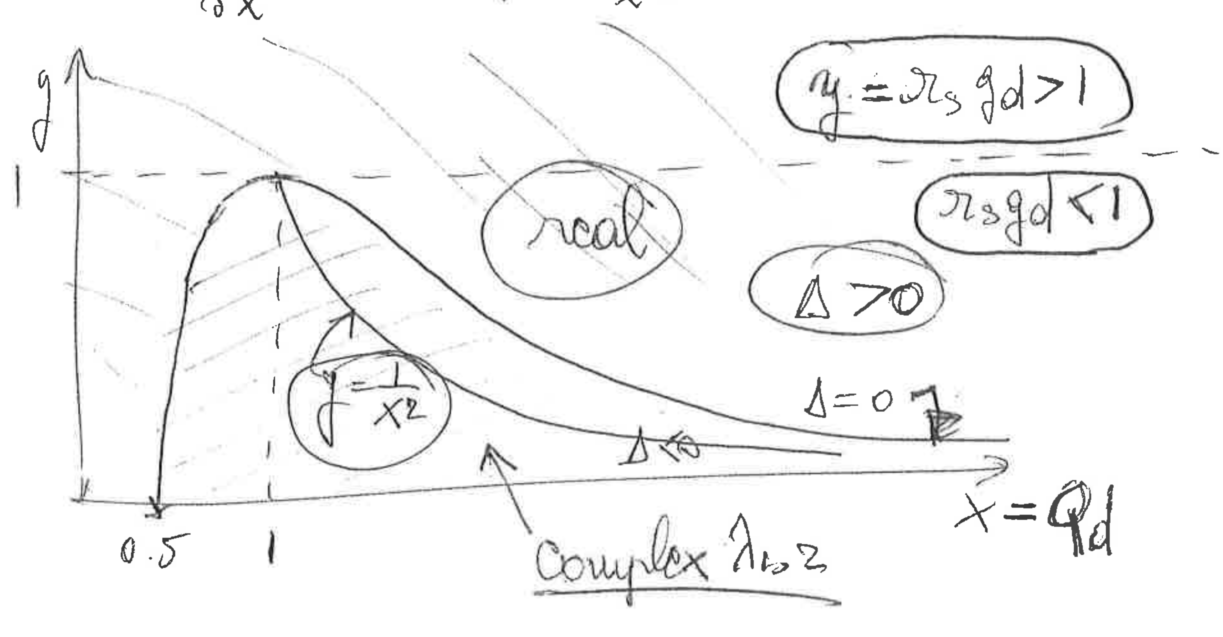
Curves (a) and (c)? $\quad xy + \frac{1}{x} - 2 = 0$

$$y = \left(2 - \frac{1}{x}\right)\frac{1}{x} = \frac{2}{x} - \frac{1}{x^2}$$

at $x = 1 \quad y = 1$

at $x = 0.5 \quad y = \frac{2}{0.5} - \frac{1}{(0.5)^2} = 4 - 4 = 0$

at $x = 1 \quad \frac{\partial y}{\partial x}$? $\quad y = -\frac{2}{x^2} + \frac{2}{x^3} = 0$ at $x = 1$



$y<1$ or $y>1$ $\rightarrow \mathrm{Re}(\lambda_{1,2})>0$

logic application

$\frac{1}{x} - xy > 0 \Longleftrightarrow y < \frac{1}{x^2}$

growing exponential

$\lambda_1, \lambda_2 > 0$

1 growing exp / 1 decay exp.
$\mathrm{Re}\,\lambda_1$, $\mathrm{Re}\,\lambda_2 < 0$

$\boxed{\begin{matrix}\mathrm{Re}\,\lambda_1 < 0 \\ \mathrm{Re}\,\lambda_2 < 0\end{matrix}}$ $\lambda_1 < 0$ $\lambda_2 < 0$ diode unequally stable

$\mathrm{Re}(\lambda_1, \lambda_2) > 0$

$\mathrm{Re}\,\lambda_1 > 0$
$\mathrm{Re}\,\lambda_2 > 0$
$\sqrt{} > 0$

$\lambda_1 > 0$
$\lambda_2 > 0$

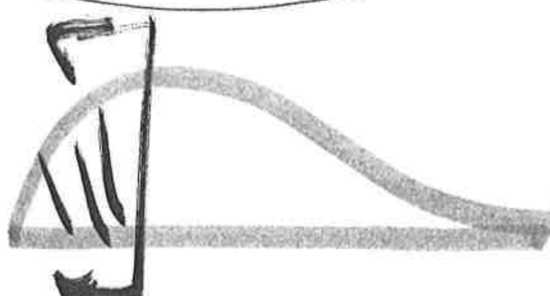
growing exponential

$\mathrm{Re}\,\lambda_1 < 0$ $\pm j\sqrt{}$
$\mathrm{Re}\,\lambda_2 < 0$

oscillations

oscillations die out exponentially

$\mathrm{Re}\,\lambda_1, \lambda_2 > 0$
$\sqrt{}$ imaginary
$\rightarrow$ oscillations grow up exponentially

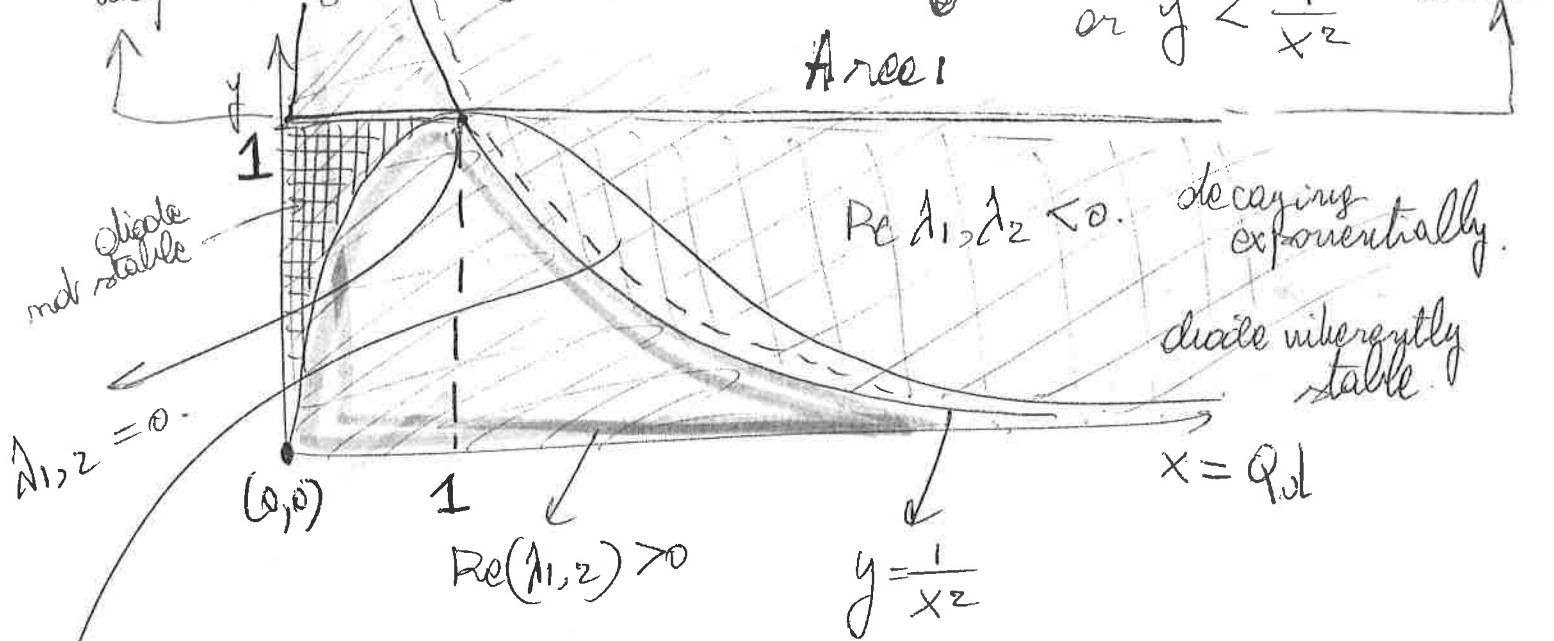


MOST Interesting Area

Are the real parts of $\lambda_{1,2}$ positive or negative

$$Re(\lambda_{1,2}) = \frac{1}{2}\left(\frac{1}{x} - xy\right) \qquad \boxed{Re(\lambda_{1,2})>0}$$

they are zero for $y = \frac{1}{x^2}$ if $\frac{1}{x} > xy$ or $y < \frac{1}{x^2}$



or Areas 1 & 2 $\sqrt{\frac{1}{4}\left(xy - \frac{1}{x}\right)^2 - (1-y)}$ is real

$$\frac{\lambda_1}{\omega_0} = \frac{1}{2}\left(\frac{1}{x} - yx\right) + \sqrt{\quad}$$

$$\frac{\lambda_2}{\omega_0} = \frac{1}{2}\left(\frac{1}{x} - yx\right) - \sqrt{\quad}$$

Equal zero at $(x,y) = (1,1)$

Oscillations die out exponentially.

$\boxed{\text{Amplifiers.}}$

oscillations grow up exponentially

How to use stability diagram

Example 1

$$Q_d = \frac{\omega_0 C_d}{g_d} \qquad \omega_0 = \frac{1}{\sqrt{L_s C_d}}$$

$|-g_d| = 30\ m℧$

$C_d = 4\,pf$

$L_s = 0.5\,nH$

$r_s = 1.5\,\Omega$

$$\boxed{\begin{array}{c} r_s g_d = 45\ 10^{-3} = y \\ \hline Q_d = 2.98 = x \end{array}}$$

root (x, y) is in Area 4 → oscillator (point 1)

We can stabilize the diode by adding small resistance r to r_s → point (x, y) is pushed in area 3.

How small r? until we are above the curve separating areas 3 & 4 i.e $y = \frac{1}{x^2}$

$$(r_s + r)\, g_d \approx \frac{1}{Q_d^2}$$

→ solve for $\underline{\underline{r}}$

Example 2 (point 2)

$|-g_d| = 150\ m℧$

$C_d = 8\,pf$

$L_s = 0.4\,nH$

$r_s = 2\,\Omega$

→ $r_s g_d = 0.312$, $Q_d = 0.906$ — we are in Area 4

small C in parallel with C_d can be added to increase Q_d. However, this additional C limits application of diode ~~to~~ as amplifier to lower frequency.

References → from Chow's book

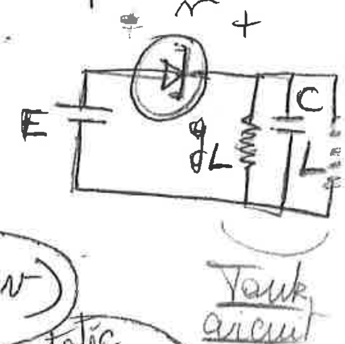
[1] R.F. Shea ed. "Principles of Transistor Circuits"
John Wiley & Sons, NY, 1953, Chapter 13.

[2] M. E. Hines "High Q Negative-Resistance Circuit Principles for Esaki Diode Applications" Bell. Syst. Tech. J., Vol. 39, p.477 (1960)

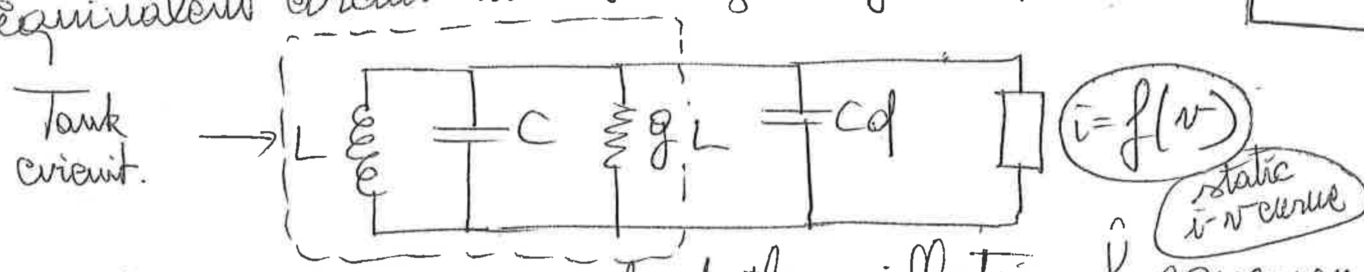
[3] Smilen, L. I and D.C. Youla "~~On the~~ stability criteria for Tunnel Diode" IRE Proc. 49, p. 1206 (1961)

Power Output Considerations

When the amplitude becomes large and swings into the nonlinear region, the effective negative conductance is reduced. The oscillation reaches its stable state when the effective, average, negative conductance balances the losses in the circuit. We want to calculate the amplitude of the oscillations under steady-state conditions.



Equivalent circuit model (neglecting L_S & r_S)



L and C + Cd are resonant at the oscillation frequency

g_L is the load conductance.

$$\sum i = 0 \quad \text{Kirchoff Law}$$

differentiate KCL with respect to t $\Rightarrow \dfrac{v}{L} + (C + C_d)\dfrac{d^2 v}{dt^2} + g_L \dfrac{dv}{dt} + \left(\dfrac{df}{dv}\right)\dfrac{dv}{dt} = 0$

$$\Rightarrow \quad C' \frac{d^2 v}{dt^2} + \left[g_L + \left(\frac{df}{dv}\right)\right]\frac{dv}{dt} + \frac{v}{L} = 0$$

This is the differential equation for the oscillator

with $C' = C + C_d$

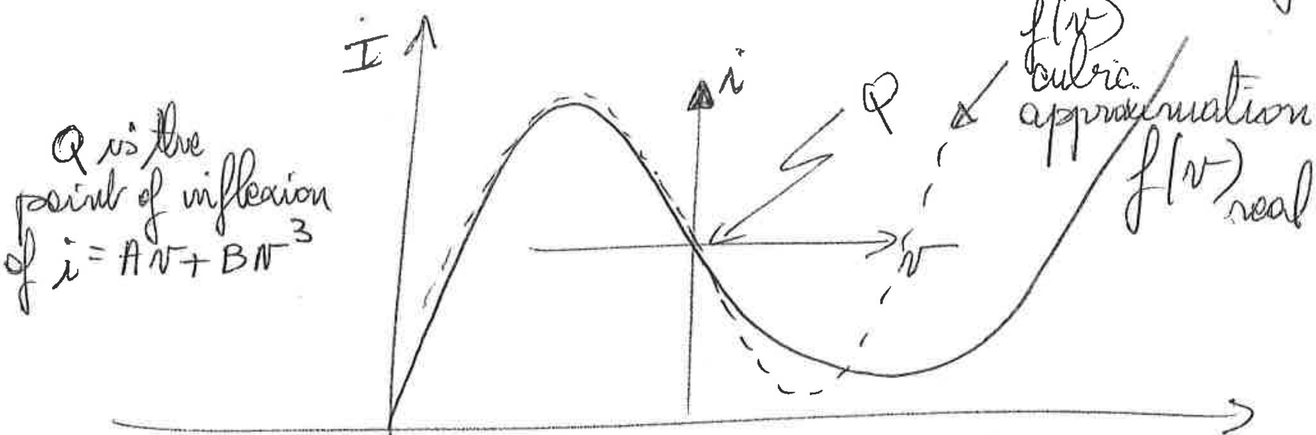
Because of the filter action of the resonant circuit, the voltage across the tank circuit is essentially sinusoidal

$$v = V \cos \omega t$$

we assume the negative conductance region can be represented by the following polynomial

$$i = f(v) = Av + Bv^3$$

where A & B are constants to be determined shortly.



Let us choose the point Q as the origin of the coordinate axes

A is obviously $-g_d$ (point Q) (B will be found later)

Substituting $v = \mathbf{V}\cos\omega t$

$i = Av + Bv^3$

in the differential equation for the oscillator and equating the average OC power generated to the average OC power dissipated in one cycle, we get

$$\frac{g_d}{2}\mathbf{V}^2 - \frac{3B}{8}\mathbf{V}^4 = \frac{g_L}{2}\mathbf{V}^2$$

$\Rightarrow$ peak oscillation amplitude

$$\Rightarrow \mathbf{V} = 2\sqrt{\frac{g_d - g_L}{3B}}$$

$$\frac{d}{dt}\Big(\underbrace{g_L v - g_d v + B v^3 + \cdots + \cdots}_{i} \Big) = 0$$

Power $\quad P = v i = g_L v^2 - g_d v^2 + B v^4$

Average over one cycle

$$\overline{P} = 0 = g_L \overline{v^2} - g_d \overline{v^2} + \overline{B v^4}$$

But ~~$\int \cos^4 \omega t$~~ $\quad \overline{P}$ means $\frac{1}{T}\int_0^T dt\, P\, dt$

$\boxed{v = V\cos\omega t}$

$$g_L \overline{v^2} = \frac{1}{T}\int_0^T \cos^2\omega t\, dt = g_L V^2 \frac{1}{T}\,\frac{T}{2} = \frac{g_L V^2}{2}$$

$$B V^4 \frac{1}{T}\int_0^T \cos^4\omega t\, dt = B V^4 \frac{1}{\omega T}\int_0^{\omega T} \cos^4 x\, dx$$

$\omega t = x$

$$= \frac{B V^4}{\omega T}\,\frac{3}{8}\,(x)\Big|_0^{\omega T}$$

$$= \frac{3}{8} B V^4$$

we used the fact that

$$\int \cos^2 x\, dx = \frac{x}{2} + \frac{\sin 2x}{4} = \frac{x}{2} + \frac{\sin x \cos x}{2}$$

$$\int \cos^4 x\, dx = \frac{3x}{8} + \frac{\sin 2x}{4} + \frac{\sin 4x}{32}$$

So $\quad (g_L - g_d)\frac{V^2}{2} + B\,\frac{3}{8} V^4 = 0$

Solving for the amplitude V

$$V = 2\sqrt{\frac{g_d - g_L}{3B}}$$

Solution possible if $g_d > g_L$ $\quad \boxed{g_d R_L > 1}$

$$\rightarrow V^2 = 4\left(\frac{g_d - g_L}{3B}\right)$$

$$V^4 = \frac{16\,(g_d - g_L)^2}{9B^2}$$

$$\rightarrow \frac{1}{2} g_L V^2 = \frac{g_d}{2}\left\{\frac{4\,(g_d - g_L)}{3B}\right\} - \frac{3B}{8}\,\frac{16}{9B^2}\left(g_d - g_L\right)^2$$

$$\frac{1}{2} g_L V^2 = \frac{2\,(g_d - g_L)}{3B}\cdot\left[g_d - (g_d - g_L)\right)$$

$$\overline{P_o} = 2(g_d - g_L)\frac{g_L}{3B} \equiv$$ Average Power to load over one cycle

$\overline{P_{o,max}}$? $\quad \frac{\partial P_o}{\partial g_L} = 0 \rightarrow g_L = \frac{g_d}{2}$

$$\overline{P_{o,max}} = \frac{g_d^2}{6B}$$

At max $\overline{P_o}$, $\quad \boxed{V_{pp} = 2V = \sqrt{\frac{8 g_d}{3B}}}$

Oscillation power delivered to load is

$$\overline{P}_o = \frac{1}{2} g_L V^2 = 2(g_d - g_L)\frac{g_L}{3B}$$

Maximum P_o (for what g_L?)

$$\frac{\partial P_o}{\partial g_L} = 0 \implies g_L = \frac{g_d}{2}$$

$$\Rightarrow \overline{P}_{o,max} = \frac{g_d^2}{6B} \quad (***)$$

$$\Rightarrow \text{at max } P_o \quad v_{pp} = 2V = \sqrt{\frac{8 g_d}{3B}}$$

But $i = f(v) = Av + Bv^3$; $A = -g_d$

peak and valley points $\left[\frac{\partial i}{\partial v}\right] = 0$

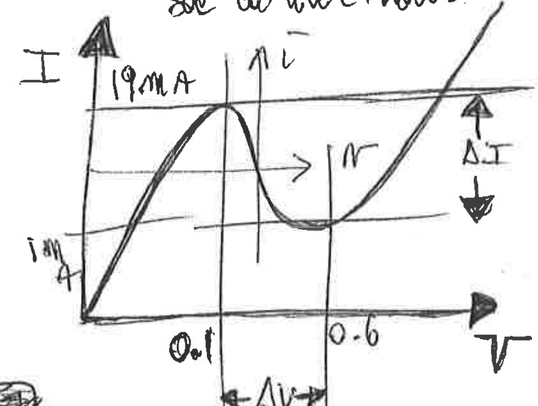
$\rightarrow$ peak $\quad v = -\sqrt{g_d/3B}$; $i = \frac{2g_d}{3}\sqrt{\frac{g_d}{3B}}$

valley $\quad v = +\sqrt{g_d/3B}$; $i = -\frac{2g_d}{3}\sqrt{\frac{g_d}{3B}}$

(**) ΔV = difference between peak & valley voltages $= 2\sqrt{\frac{g_d}{3B}}$

(*) ΔI = difference between peak & valley currents $= \frac{4g_d}{3}\sqrt{\frac{g_d}{3B}}$

See earlier notes



Divide (*) by (**) $\Longrightarrow$ $\boxed{g_d = \frac{3}{2}\frac{\Delta I}{\Delta V}}$

Use (**) to Solve for B $\quad \boxed{B = 2\frac{\Delta I}{(\Delta V)^3}}$

Plug g_d & B in Eq. (***) Above $\Longrightarrow$ $\boxed{P_{o,max} = \frac{3}{16}\Delta V \Delta I}$ $\rightarrow$ $\boxed{P_{max} \simeq 2\,mW}$

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Returning to

$$\frac{g_d}{2} V^2 - \frac{3B}{8} V^4 = \frac{g_L}{2} V^2$$

$\Rightarrow$ under steady state oscillations, the average negative conductance is given by

$$g_{av} = -g_d + \frac{3B}{4} V^2$$

$\boxed{\text{we want } \frac{1}{2} g_{av} V^2 + \frac{1}{2} g_L V^2 = 0}$

For the case of $P_o = P_{o,max}$

$$P_{o,max} = \frac{g_d^2}{6B} = \frac{9}{4}\left(\frac{\Delta I}{\Delta V}\right)^2 \frac{(\Delta V)^3}{6 \cdot 2\, \Delta I}$$

$$P_{o,max} = \frac{3}{16} \Delta V \Delta I$$

$\boxed{g_d = \frac{3}{2}\left(\frac{\Delta I}{\Delta V}\right)}$

$$V = \frac{1}{2}\sqrt{\frac{8 g_d}{3B}}$$

$$\Rightarrow g_{av} = -g_d + \frac{\cancel{3B}}{4} \frac{1}{4} \frac{8 g_d}{\cancel{3B}}$$

$$\boxed{g_{av} = -\frac{g_d}{2}}$$

Reference: Chow's book pp 167-170.

Chow, "Principles of Tunnel Diodes", on TK 7872.D6C5