8. (30 pts) For the NMOS amplifier below, replace the transistor with its T equivalent circuit while neglecting the effect of $r_0$. Calculate the voltage gains $v_S/v_i$ and $v_d/v_i$.

\[
\frac{v_S}{v_i} = \frac{R_s}{R_s + \frac{1}{g_m}} = \frac{g_m R_s}{1 + g_m R_s}
\]

\[
\frac{v_d}{v_i} = -g_m \cdot V_{GS} \cdot R_D
\]

\[
V_{GS} = \frac{1}{g_m} \left( \frac{1}{R_s} + v_i \right) = \frac{v_i}{1 + g_m R_s}
\]

\[
\text{So, } \frac{v_d}{v_i} = -\frac{g_m R_D}{1 + g_m R_s}
\]
V. For the circuit shown below, both transistors are characterized by the following parameters $\mu_n C_{ox} = 20 \mu A/V^2$, $V_t = 1V$, $\lambda = 0$, $L = 10 \mu m$, and $W = 30 \mu m$. Find the labeled current and voltage.

\[ I_2 = \frac{1}{2} \times 30 \times \frac{30}{10} (15-1)^2 = 7.5 \mu A \]
7. (30 pts) For the amplifier circuit shown below:

- Which type of amplifier configuration is it? common source, common gate, or common drain. Circle your answer.
- Use the hybrid-π equivalent AC circuit for the transistor and draw the small signal equivalent circuit for the amplifier.
- Give an analytical expression for the input resistance $R_{in}$, as shown on the diagram.
- Give an analytical expression for the overall voltage gain $v_0/v_{sig}$.

\[ R_{in} = \left( \frac{R_1}{R_2} \right) + R_m \]
\[ V_{0} = -g_m N_{gs} \left( \frac{R_D}{R_L} \right) \]
\[ N_{gs} = \frac{R_{in}}{R_{in} + \left( \frac{R_1}{R_2} \right)} \]
\[ N_{sig} = \frac{g_m N_{gs} R_{sig} + \left( \frac{R_1}{R_2} \right)}{R_{sig} + \left( \frac{R_1}{R_2} \right)} \]
\[ N_{0} = -g_m \left( \frac{R_D}{R_L} \right) \left( \frac{R_1}{R_2} \right) \]
6. (20 pts) The transistor in the circuit below has a $V_t = 1V$, $k_n' \left( \frac{W}{L} \right) = 0.4mA/V^2$ and $\lambda = 0$.

Find the labeled node voltage $V_1$.

\[ \frac{V_1 + 5}{100} = \frac{1}{2} 0.4 \left( 5 - \frac{100 I}{V_1} \right)^2 + 5 \]

\[ I = \frac{0.035}{0.036} \text{ mA} \]

Because $V_{ES} < V_T \rightarrow \text{cut-off}$

Only acceptable solution

\[ V_1 = -5 + 100 I = -1.4V \]

\[ I = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_T)^2 \]

\[ = \frac{1}{2} k_n' \frac{W}{L} (0 - V_T)^2 \]

\[ = \frac{1}{2} k_n' \frac{W}{L} (-(-5 + 100 I) - 1)^2 \]

\[ I = 0.2 (4 - 100 I)^2 \rightarrow I_1, I_2 \text{ above} \]
5. (20 pts) The NMOS transistors in the circuit below have $V_t = 1V$, $k_n \frac{W}{L} = 2mA/V^2$ and $\lambda = 0$.

Find the labeled node voltages, $V_1$, $V_2$, and $V_3$.

\[
\begin{align*}
10 - V_3 - V_1 &= I_0 \\
I_{D1} &= \frac{V_1}{1} = \frac{1}{2} \cdot 2 (V_3 - V_2 - 1)^2 \\
I_{D2} &= \frac{V_1}{1} = \frac{1}{2} \cdot 2 (V_2 - V_1 - 1)^2 \\
V_3 - V_2 - 1 &= V_2 - V_1 - 1 \\
V_1 &= 2V_2 - V_3 \\
V_2 - V_3 &= 60 - V_3 \\
V_2 &= 5V \\
V_1 &= (V_1 - V_1)^2 \\
V_1 &= 6.85 \text{ or } 2.45V \\
V_3 &= 10 - 2.45 \text{ mA (1 k\Omega)} = 7.55V
\end{align*}
\]
VI. The MOSFET in the circuit below has $V_t = 1V$, $K = 0.4mA/V^2$, and $V_A = 40V$. (a) Assuming $R_D = 50k\Omega$, find $R_S$ and $R_G$ so that $I_D = 0.1mA$. (b) Find the values of $g_m$ and $r_0$ at the bias point. (c) If terminal $Y$ is grounded, find the voltage gain from $X$ to $Z$ with $Z$ open-circuited. (d) If terminal $X$ is grounded and terminal $Z$ is connected to a current source delivering a current of $10\mu A$ and having a resistance of $100$ k$\Omega$, find the voltage signal at $Y$. Neglect $r_0$ in that case.

\[ g_m = 2K(V_{GS} - V_t) = 0.4 \text{mA/V}^2 \]
\[ r_0 = \frac{V_A}{I_D} = 400k\Omega + 2 \]

\[ R_S \text{ undetermined, but must be high} \]
\[ R_G = \frac{V_D}{0+2} \]
\[ +2R_D = \frac{50k\Omega}{0.1} = 500k\Omega \]
\[ 0.1 = 0.4(V_{GS} - 1)^2 \]
\[ V_{GS} = 1.5V + 2 \]
\[ R_S = \frac{-1.5 - (-5)}{0.1} = 35k\Omega \]

\[ A_{in} = \frac{(R_S/\| r_0)}{(R_S/\| r_0) + \frac{1}{g_m}} = 0.928 \text{V/V} + 2 \]

\[ i_D = i_C \frac{(R/\| R_S) + 0.4}{(R/\| R_S) + 0.4 + g_m} \]
\[ = 10 \frac{(100/35) + 0.4}{(100/35) + 0.4} = 9.12 mA \]
\[ V_0 = R_D i_D = 9.12 \times 50 = 456 \text{mV} \]
8. (25 pts) The amplifier circuit shown below is biased in the saturation mode of operation with a value of the transconductance $g_m = 1 \text{ mA/V}$. Assume $\lambda = 0$.

- Use the hybrid-$\pi$ model of the transistor and draw the small signal equivalent circuit for the amplifier.
- Calculate the voltage gain $v_o/v_{\text{sig}}$.

\[ v_o = -g_m (R_{D//R_L}) \left( \frac{47/10}{47/10 + 0.1} \right) v_{\text{sig}} \]

\[ \frac{v_o}{v_{\text{sig}}} = -1 \left( \frac{4.7k/10k}{8.25} \right) \left( \frac{8.25}{8.25 + 0.1} \right) \]

\[ v_o = -3.16V/V \]
6. (25 pts) In the circuit below, the NMOS has a $|V_i|$ of $0.9$ V and a $V_A$ of $50$ V and operates with $V_D = 2V$.

(a) Using the hybrid-$\pi$ model of the MOS transistor, draw the small signal equivalent circuit of the amplifier. Include the effect of $r_0$.

(b) What do the DC value $V_D$ becomes if the current $I$ is changed to $1$ mA? Neglect the effects of $V_A$ in the expression of $I_D$. 

$$I = 500 \mu A$$

$$R_C = 10 \Omega$$

$$R_L = 10 \Omega$$

$$V_L = 5V$$

$$V_D = 2V$$

\[ \begin{align*}
I_D &= \frac{k_m}{2} \frac{W}{L} (V_{GS1} - V_T)^2 \\
I_D &= \frac{k_m}{2} \frac{W}{L} (V_{GS2} - V_T)^2 \\
I_D &= \frac{(V_{GS1} - V_T)^2}{(V_{GS2} - V_T)^2} \\
\Rightarrow \frac{I_D1}{I_D2} &= \frac{(V_{GS1} - V_T)^2}{(V_{GS2} - V_T)^2} \\
V_{GS2} &= V_T + \sqrt{2} (V_{GS1} - V_T) \\
V_{GS2} &= 0.9 + \sqrt{2}(2 - 0.9) = 2.5V
\end{align*} \]
5. (25 pts) The NMOS transistors in the circuit below have $V_I = 1 V$, $\mu_n C_{ox} = 120 \mu A / V^2$, $\lambda = 0$, and $L_1, L_2, L_3 = 1 \mu m$.

Find the required values of the gate widths $W_1$ of $Q_1$, $W_2$ of $Q_2$, $W_3$ of $Q_3$ to obtain the voltage and current values indicated on the figure.

\[ I_D = \frac{1}{2} k^* \frac{W}{L} \left( \frac{V_{GS}}{V_T} \right)^2 \]

\[ k^* = \mu_n C_{ox} \]

\[ V_{GS1} = 1.5 V \quad \rightarrow \quad 120 \mu A = \frac{1}{2} \cdot 120 \cdot \frac{W_1}{L_1} (1.5 - 1)^2 \quad \Rightarrow \quad W_1 = 8 \mu m \]

\[ V_{GS2} = 3.5 - 1.5 = 2 V \quad \rightarrow \quad 120 = \frac{1}{2} \cdot 120 \cdot \frac{W_2}{L_2} (2 - 1)^2 \quad \Rightarrow \quad W_2 = 2 \mu m \]

\[ V_{GS3} = 1.5 V \quad \rightarrow \quad W_3 = W_1 = 8 \mu m \]