Deutsch–Jozsa Algorithm

Alice, in Amsterdam, selects a number $x$ from 0 to $2^n - 1$, and mails it in a letter to Bob, in Boston. Each $x$ has $n$ bits of information.

Bob calculates some function $f(x)$ and replies with the result, which is either 0 or 1.

Bob has promised to use a function of 2 kinds:
- Either $f(x)$ is constant for $x$, or $f(x)$ is balanced that is equal to 1 for exactly half of all possible $x$, and 0 for the other half.

Alice's goal is to determine with certainty whether Bob has chosen a constant or balanced function, corresponding with him as little as possible.

How fast can she succeed?
Classically

Alice may only send one value of \( x \) \( n \) each letter. At worst, Alice will need to query Bob \( 2^n + 1 \) times, since she may receive \( 2^{m/2} \) \( \Theta(2^n) \) before finally getting a \( \Theta(1) \) (\( e \)).

Telling her that Bob's function is balanced.

The best deterministic algorithm she can use therefore requires \( 2^{m/2} + 1 \) queries.

Quantum-mechanically, Alice can achieve her goal in just one correspondence.