Deutsch-Jozsa Algorithm

Alice, in Amsterdam, selects a number x from 0 to $2^n - 1$, and mails it in a letter to Bob, in Boston.

↳ each x has n bits of information

Bob calculates some function $f(x)$ and replies with the result, which is either 0 or 1.

Bob has promised to use a function of 2 kinds. Either $f(x)$ is constant $\forall x$, or $f(x)$ is balanced. That is equal to 1 for exactly half of all possible x, and 0 for the other half.

Alice's goal is to determine with certainty whether Bob has chosen a constant or balanced function, corresponding with him as little as possible.

How fast can she succeed?

Classically

Alice may only send one value of x at each letter. At worst, Alice will need to query Bob $\frac{2^n}{2}+1$ times, since she may received $\frac{2^n}{2}$ 0s (or 1s) before finally getting a 1 (or 0). telling her that Bob's function is balanced.

The best deterministic algorithm she can use therefore requires $2^n/2+1$ queries.

Quantum-mechanically, Alice can achieve her goal in just one correspondence.