Ex: 5.1

\[ C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} = \frac{34.5 \text{ pF/m}}{4 \text{ nm}} = 8.625 \text{ fF/} (\text{\mu m})^2 \]

\[ \mu_n = 450 \text{ cm}^2/\text{VS} \]

\[ k'_n = \mu_n C_{ox} = 388 \text{ \mu A/} V^2 \]

\[ V_{OV} = (V_{GS} - V_t) = 0.5 \text{ V} \]

\[ g_{DS} = \frac{1}{1 \text{ k}\Omega} = k'_n \frac{W}{L} V_{OV} \Rightarrow \frac{W}{L} = 5.15 \]

\[ L = 0.18 \text{ \mu m, so } W = 0.93 \text{ \mu m} \]

Ex: 5.3 \[ I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 \text{ in saturation} \]

Change in \( I_D \) is:
(a) double \( L \), 0.5
(b) double \( W \), 2
(c) double \( V_{OV} \), \( 2^2 = 4 \)
(d) double \( V_{DS} \), no change (ignoring length modulation)
(e) changes (a) - (d), 4

Case (c) would cause leaving saturation if
\[ V_{DS} < 2V_{OV} \]
Ex: 5.5 \( V_{OV} = 0.5 \) V

\[ g_{DS} = k_n \frac{W}{L} V_{OV} = \frac{1}{1 \text{ k}\Omega} \]

\[ k_n = k'_n \frac{W}{L} = 2 \text{ mA} / \text{V}^2 \]

For \( v_{DS} = 0.5 \) V \( \cdot \) \( = V_{OV} \)

\[ I_D = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2 = 0.25 \text{ mA} \]

for all \( v_{DS} \geq V_{OV} = 0.5 \) V.

---

Ex: 5.9

\[ \begin{align*}
W &= 0.72 \mu\text{m} = 4.0 \\
L &= 0.18 \mu\text{m} \\
\lambda &= 0 \\
\text{saturation mode} \quad (v_{GD} = 0 < V_{th})
\end{align*} \]

\[ 
V_D = 0.8 \text{ V.} = 1.8 - I_D R_D \\
I_D = \frac{1}{2} \mu_n C_{ov} \frac{W}{L} (V_D - V_{th})^2 = 72 \mu\text{A} \\
\therefore R = \frac{1.8 - 0.8}{72 \mu\text{A}} = 13.9 \text{ k}\Omega
\]
\[ V_t = 0.7 \text{ V.} \]
\[ k_n = 1 \text{ mA/V}^2 \]

\[
\begin{align*}
V_i &= v_G - v_s \\
\text{Design for } A_v &= \frac{v_G}{v_i} = -25, \quad R_{in} = 500 \text{ k}\Omega \\
\therefore g_m R_D &= 25 = k_n V_{OV} R_D \\
R_{in} &= \frac{v_i}{ii} = \frac{v_i}{v_i - v_n} R_G \\
\Rightarrow R_G &= 26 R_{in} = 13 \text{ M}\Omega \\
I_D R_D &= \left( \frac{1}{2} k_n V_{OV}^2 \right) R_D \\
&= \frac{1}{2} g_m R_D V_{OV} = 12.5 \text{ V} \\
\text{and } V_{OV} &= V_{DD} - V_t - I_D R_D = 4.3 - 12.5 \text{ V} \\
\therefore V_{OV} &= 0.319 \text{ V.} \\
g_m &= 319 \text{ \mu A/V} \\
R_D &= 78.5 \text{ k}\Omega \\
V_{DS} &= V_{OV} + V_t \\
\Delta V_{GD} &= 0 + 26 \Delta v_t \leq V_t \\
\therefore |\Delta v_t| &< \frac{V_t}{26} = 27 \text{ mV.} 
\end{align*}
\]
Ex: 5.24

\[ I_D = \frac{1}{2} k_n \frac{W}{L} (V_{SG} - |V_i|)^2 \]

\[ = \frac{1}{2} \times 60 \times \frac{16}{0.8} \times (1.6 - 1)^2 \]

\[ I_D = 216 \text{ \(\mu\)A} \]

\[ g_m = \frac{2I_D}{V_{OV}} = \frac{2 \times 216}{1.6 - 1} = 720 \text{ \(\mu\)A/V} \]

\[ = 0.72 \text{ mA/V} \]

\[ \lambda = 0.04 \Rightarrow V_A' = \frac{1}{\lambda} = \frac{1}{0.04} = 25 \text{ V/\mu m} \]

\[ r_o = \frac{V_A' \times L}{I_D} = \frac{25 \times 0.8}{0.216} = 92.6 \text{ k\Omega} \]

Ex: 5.29

\[ R_{in} = \frac{1}{g_m} = R_{sig} = 100 \text{ \Omega} \]

\[ \Rightarrow g_m = 10 \text{ mA/V} \]

\[ g_m = \frac{2I_D}{V_{OV}} = \frac{2I_D}{0.2V} \Rightarrow I_D = 1 \text{ mA} \]

\[ G_V = \frac{v_o}{v_{sig}} = \frac{R_{in}}{R_{sig} + R_{in} g_m R_D} \]

\[ = \left( \frac{1}{2} \right) (10 \text{ mA/V})(2 \text{ k\Omega}) \]

\[ = +10 \]
\( g_m = 1 \text{ mA} / \text{V} \)

For \( R_{\text{sig}} = 50 \Omega \)

\( k_{\text{in}} = \frac{1}{g_m} = 1 \text{ k}\Omega \)

\( k_{\text{out}} = R_D = 15 \text{ k}\Omega \)

\( A_{V_{\text{in}}} = +g_mR_D = +15 \)

\( A_V = g_m(R_D \parallel R_L) = +7.5 \)

\( G_V = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}}A_V = 7.1 \)

\( k_n = 0.4 \text{mA} / \text{V}^2 \)

\( V_t = 0.5 \text{ V} \)

\( \lambda = 0 \)

sat. boundary \( V_{GD} = 0.5 \text{V} , = I_D R_D \)

\( 0.5 \text{V} , = \frac{1}{2}k_n \frac{W}{L}(1.8 - 0.5)^2R_D \)

\( \therefore \frac{W}{L}R_D = 1.48 \text{ k}\Omega \)
\( V_T = -0.6 \, \text{V} \)

\( k_n = 100 \, \mu \text{A} / \text{V}^2 \)

\( L = 0.25 \, \mu \text{m} \)

\( \lambda = 0 \)

make \( i_D = 0.8 \, \text{mA} \), \( V_D = 1.5 \, \text{V} \).

\[
R = \frac{1.5 \, \text{V}}{0.8 \, \text{mA}} = 1.875 \, \text{k\Omega} \quad V_{S0} = +1 \, \text{V}.
\]

\[
0.8 \, \text{mA} = \frac{1}{2} k_n \cdot \frac{W}{L} (1 - 0.6)^2.
\]

\[
\frac{W}{L} = 100.
\]

\( W = 25 \, \mu \text{m} \)
5.56 All parts $k_s = 0.5 \frac{mA}{V^2}$, $V_t = 0.8 V$.

(a) 

$$ V_i = -V_{gs} = -\sqrt{\frac{2 \times (10 \ \mu A)}{0.5 \ mA/V^2}} - 0.8 V. $$

$$ = -1 \ V. $$

(b) same as (a), except $i_D = 100 \ \mu A$

$$ V_2 = -1.432 \ V, $$

(c) same as (a), except $i_D = 1 \ mA$

$$ V_3 = -2.80 \ V, $$

$$ V_s = -V_{gs} = -5 + 100K \ I_D $$

$$ V_{gs} = 5 - (100 \left(\frac{1}{2}\right)(0.5) \left[V_{gs} - 0.8\right]^2 $$

$$ 0 = 5 - 25(V_{gs} - 0.8)^2 - V_{gs} $$

see soln. in (f)

$$ V_{gs} = +1.19 \ V. \quad I_D = 38.1 \ \mu A $$

$$ V_s = -V_{gs} = -1.19 \ V. $$
Both $Q_1$ and $Q_2$, in sat.

($V_{GD1} = V_{GD2} = 0$)

:. both $Q_1$ and $Q_2$ have same $V_{GS}$

\[ +5 - 1KI_D - V_{GS} - V_{GS} - 1KI_D = 0 \]

\[ 5 - (2)\left(\frac{1}{2}\right)[5][V_{GS} - 1]^2 - 2V_{GS} = 0 \]

\[ 0 = -5V_{GS}^2 + 8V_{GS} \]

\[ V_{GS} = +1.60\text{V}, \text{ (bad root < } V_i) \]

\[ I_D = 0.90\text{mA} = \frac{1}{2}\left(\frac{5\text{mA}}{V^2}\right) [1.6 - 1]^2 \]

\[ V_1 = +5 - (1k)I_D = +4.1\text{V}. \]

\[ V_2 = V_1 - V_{GS} = +2.5\text{V}. \]

\[ V_3 = V_2 - V_{GS} = (1k)I_D = 0.9\text{ V} \]
5.76 continues here

\[ V_i = (g_m v_{gs}) \left( \frac{1}{g_m} + R_S \right) \]

\[ v_d = -g_m v_{gs} R_D \]

\[ v_S = +g_m v_{gs} R_S \]

\[ \frac{v_S}{V_i} = \frac{R_S}{1 + g_m R_S} = \frac{g_m R_S}{1 + g_m R_S} \]

\[ \frac{v_d}{V_i} = -\frac{R_D}{1 + R_S} = -\frac{g_m R_D}{1 + g_m R_S} \]

5.79 \[ V_i = 1 \text{V}, \quad k_i = \frac{W}{L} = 2 \text{mA/V}^2 \]

(a) dc analysis \[ V_G = \frac{5}{15} 15 \text{V} = 5 \text{V} \text{, assume} \]

\[ I_D = 1 \text{mA} \]

\[ V_S = 3 \text{V}, \quad V_{GS} = 2 \text{V}, \quad V_0V = 1 \text{V}. \]

\[ I_D = \frac{1}{2} k' v_0^2 = 1 \text{mA (check)} \]
(b) \( r_0 = \frac{V_O}{I_D} = \frac{100 \text{ V}}{1 \text{ mA}} = 100 \text{ k}\Omega \)

\( r_m = \sqrt{2\Delta I_D} = 2 \text{ mS} \)

(c)

\[
\begin{align*}
R_{\text{in}} & = 100 \text{ k}\Omega \\
R_G & = 3.33 \text{ m}\Omega \\
R_D & = 7.5 \text{ k}\Omega \\
R_L & = 10 \text{ k}\Omega \\
g_m & = -8.2 \\
v_o & = -8.0 \\
\end{align*}
\]

(d) \( R_{\text{in}} = R_G = 3.33 \text{ M}\Omega \)

\[
\frac{v_o}{v_{in}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{ex}}} = 0.97
\]

\[
\frac{v_o}{v_{ex}} = -g_m (r_0 [R_G R_I]) = -8.2
\]

\[
\frac{v_o}{v_{Sig}} = -8.0
\]

5.88

For \( Q_1 \) and \( Q_2 \): \( I_D = 0.25 \text{ mA} \)

\[
V_{ov} = 0.25 \text{ V} \Rightarrow g_{m1, 2} = \frac{2 \cdot 0.25 \text{ mA}}{0.25 \text{ V}} = 2 \text{ mA/V}
\]

\[
V_A \text{ very large} \Rightarrow r_{o1, 2} = \infty
\]
b) Overall \( G_V = G_{V1} \cdot G_{V2} \)

\[
G_{V1} = -g_{m1} \cdot (\infty \parallel R_D)
\]

\[
= \frac{-2 \text{ mA}}{V} \cdot 10 \text{ k}\Omega = -20 \text{ V/V}
\]

\[
G_{V2} = -g_{m2} \cdot (\infty \parallel R_D \parallel R_L)
\]

\[
= \frac{-2 \text{ mA}}{V} \cdot (10 \text{ K} \parallel 10 \text{ K})\Omega = -10 \text{ V/V}
\]

\[
\Rightarrow G_V = (-20) \cdot (-10) = 200 \text{ V/V}
\]

5.90 \( g_m = 5 \text{ mS} \)

\[
i_d = g_m v_{gs} = \frac{g_m}{1 + g_m R_s} v_g
\]

\[
\frac{g_m}{1 + g_m R_s} = 1 \text{ mS}
\]

\[
\therefore R_s = \frac{4}{g_m} = 800 \Omega
\]
\[ V_{ip} = -0.7 \text{V}, \quad V_{A} \rightarrow \infty \]

a) for \( I_D = 0.3 \text{ mA}, |V_{OV}| = 0.3 \text{ V}. \)

\[ V_{SG} = 1.0 \text{ V}, \quad V_G = 0 \]

\[ V_S = 2.5 - I_D R_S = 1.0 \text{ V}. \]

\[ \therefore R_S = 5.0 \text{ k}\Omega \]

b) \( g_m = \frac{2I_D}{V_{OV}} = 2 \text{ mS} \)

\[ G_V = \frac{v_0}{v_{sig}} = -g_m R_D = -10 \]

\[ \therefore R_D = 5.0 \text{ k}\Omega \]

c) \( v_{gd} + V_{GD} \geq V_{ip} = -0.7 \)

\[ -\left| \hat{v}_o + \frac{\hat{v}_o}{10} \right| + 1 \text{V} \geq -0.7 \]

\[ \hat{v}_o \leq 1.55 \text{V pk} \]

\[ \hat{v}_{sig} \leq \frac{\hat{v}_o \text{ max}}{10} = 0.155 \text{V pk} \]

d) for \( \hat{v}_{sig} = 50 \text{ mV}, \) changed \( R_D \)

\[ -\left| 1 + \frac{\hat{v}_o}{g_m R_D} \right| + (2.5 - I_D R_D) \geq -0.7 \]

for \( g_m = 2 \text{ mS}, I_D = 0.3 \text{ mA} \)

\[ -\left| \frac{1 + g_m R_D}{g_m R_D} \right| g_m R_D \hat{v}_{sig} + 2.5 - I_D R_D \geq -0.7 \]

\[ R_D \leq 7.88 \text{ k}\Omega \quad (\hat{v}_{sig} = 50 \text{ mV}) \]

\[ G_V = -g_m R_D = -15.8 \]
\[ V_i = 1 \text{V.} \quad k_n = 0.8 \text{mA/V}^2 \]

\[ V_A = 40 \text{V.} \quad I_D = 0.1 \text{mA} \]

a) \( R_G = 10 \ \text{M}\Omega \)

\[ V_{OV} = \sqrt{\frac{2I_D}{k_n}} = 0.5 \text{V.} \]

\[ V_{GS} = 1.5 \text{V.} \quad R_S = 35k\Omega \]

for \( V_D = V_S + V_O + 1 \text{V.} = 0 \text{V.} \)

\( R_D = 50k\Omega \)

b) \( g_m = \frac{2I_D}{V_{OV}} = 0.4 \text{mS} \)

\[ r_0 = \frac{V_A}{I_D} = 400 \ \text{k}\Omega \]

c) \[ G_V = \frac{\nu_v}{\nu_{sig}} = -\frac{10 \text{M}}{1 \text{M} + 10 \text{M} g_m} \]

\[ (r_0 \parallel R_D \parallel 40 \text{K}) = -7.7 \]
\[ \frac{v_y}{v_x} = \frac{g_m (R_0 \parallel r_0)}{1 + g_m (R_0 \parallel r_0)} = 0.93 \]

\[ R_{\text{out}} = \frac{1}{g_m} R_i \parallel r_0 \]

\[ = 2.32 \text{ k}\Omega \]

c)

\[ v_i = i_{ug} \left[ \frac{1}{g_m} R_{ug} \parallel R_s \right] g_m R_D \]

\[ = \left( 45.6 \text{ k}\Omega \right) i_{ug} \]

\[ = 0.456V. \]