EECE 250

NETWORK ANALYSIS I

Course syllabus

Prerequisite : Physics III
Corequisite : Differential Equations

Prerequisite for : Network Analysis II
and ----

Textbook — Drill Problems

Problem session — homework problems

Course grade

2 Hour exams : 25% each

1 Final exam : 30 %

Homework : 10 %

Quizzes : 10%
+ PSpice

Chapter 2 - Definitions & Units

International System of Units
("Metric" or "SI" or "MKS")

Arbitrarily defined units:

Length: meter (m)
Mass: kilogram (kg)
Time: second (s)

Derived units:

Force: newton (N) $F = ma$
Work and energy: joule (J) $E = mc^2$
Power: watt (W) $P = VI$

Consistency of unit system

SI (International system of units) was adopted in 1960.

Seven basic units:

metric system:
- meter
- kilogram
- second
- ampere
- kelvin
- mole (amount of substance)
- candella (luminous intensity)

Units for other quantities (volume, force, energy ...) are derived from these seven basic units.

$$\text{calorie} \leftrightarrow \left(\overset{\text{Food}}{\downarrow}\ 1\,\text{kcalorie}\right) = 4.187\,J.$$

- $1\ \text{Joule} = 1\ kg\ m^2 s^{-2}$

- kilowatthour (kWh)

$$1h = 3{,}600\,s$$

$$kWh = 3.6\ 10^6\ J$$

- The fundamental unit of power is the watt (W)

$$1W = 1\,J/s$$

SI uses the decimal system to relate larger and smaller units to the basic unit.

Other arbitrarily defined units:

Electric current: ampere (A)
Temperature: kelvin (K)
Luminous intensity: candela (cd)

Scientific and <u>engineering</u> notation:

$$0.0247\ m = 2.47\times10^{-2}\ m = \underline{24.7\times10^{-3}\ m}$$

<u>Prefixes</u>:

(femto)	f	10^{-15}			
(pico)	p	10^{-12}	(centi)	c	10^{-2}
			(deci)	d	10^{-1}
(nano)	n	10^{-9}	(kilo)	k	10^{3}
(micro)	μ	10^{-6}	(mega)	M	10^{6}
(milli)	m	10^{-3}	(giga)	G	10^{9}
			(tera)	T	10^{12}

$$0.0247\ m = 24.7\ mm$$

(hecto)	h	10^{2}
(deka)	da	10^{1}

Nanotechnology

$1 Å = 10^{-10} m$

$nm = 10 Å = 10^{-9} m$

Expressing numbers in "engineering units"

In engineering notation, a quantity is represented by a number between 1 and 999 and an appropriate metric unit using a power divisible by 3.

Example $0.048 W \longrightarrow$ 48 mW instead of 4.8 cW or $4.8\ 10^{-2} W$ or 48,000 μW

Practice

2.1 A krypton fluoride (KF) laser emits light at a wavelength of 248 nm

This is the same as
(a) 0.0248 mm
(b) 2.48 μm
(c) 0.248 μm
(d) 24,800 Å

2.1

Do practice problems 2.2 & 2.3 on page 11

EXAMPLE

Convert the following to engineering notation

a) 1.2×10^{-5} s → 12 μs (10^{-6} s)

b) 750 mJ → 750 mJ

c) 1130 Ω → 1.13 kΩ

d) 3,500,000,000 bits → 3.5 Gb (10^{9})

e) 0.0065 μm → $6.5 \underbrace{10^{-9}\,m}_{nm} = 6.5\,nm$

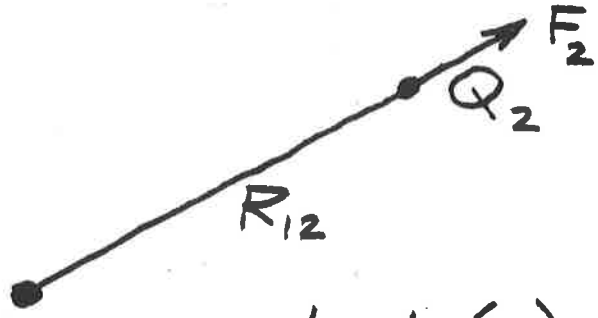
f) 13,560,000 Hz → 13.56 MHz (10^{6})

g) 0.039 nA → $39 \underbrace{10^{-12}\,A}_{pA} = 39\,pA$

h) 49,000 Ω → 49 kΩ

i) 1.173×10^{-5} μA → $11.73 \underbrace{10^{-12}\,A}_{pA} = 11.73\,pA$

2.2 Electric charge & Coulomb's Law



$$F_2 \propto \frac{Q_1 Q_2}{R_{12}^2}$$

$$F_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}$$

$Q_1 \leftarrow$ coulomb (C)

$\epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m

In SI system, the fundamental unit of charge is the Coulomb

For an electron:

$$q = -1.602 \times 10^{-19} \text{ C}$$

Comparisons to mass:

$$F_2 = G \frac{M_1 M_2}{R_{12}^2}$$

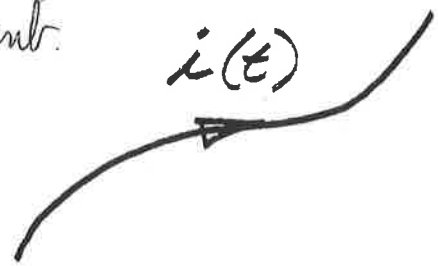
$\varepsilon_0 = 8.854 \; 10^{-12}$ F/m

permittivity of free space.

or dielectric constant.

2.2 Electric current (p12)

charge in motion represents a current.



i = time rate of flow of (positive) charge through a surface (C/s or ampere (A))

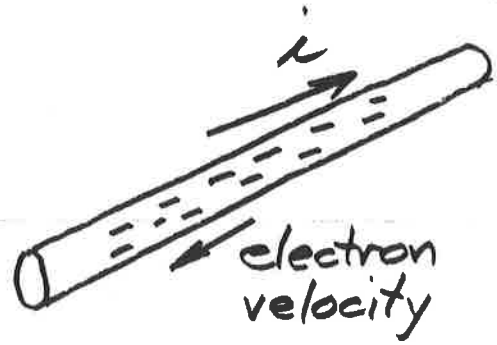
$$i = \frac{q}{t} \quad \text{or} \quad \frac{\Delta q}{\Delta t} \quad \text{or} \quad \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$$

$$= \frac{dq}{dt}$$

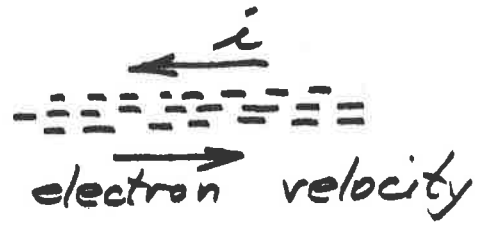
[1A corresponds to 1 Coulomb of charge passing through an arbitrary chosen cross section in one second]

Examples:

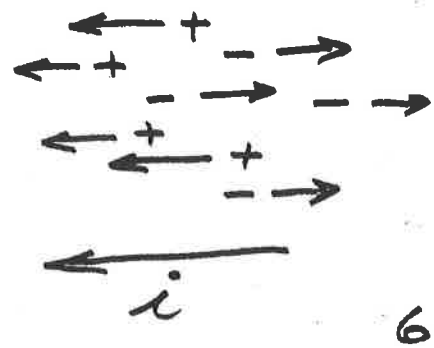
conduction electrons in copper wire



convection electrons in CRT



electrons and holes in semiconductor



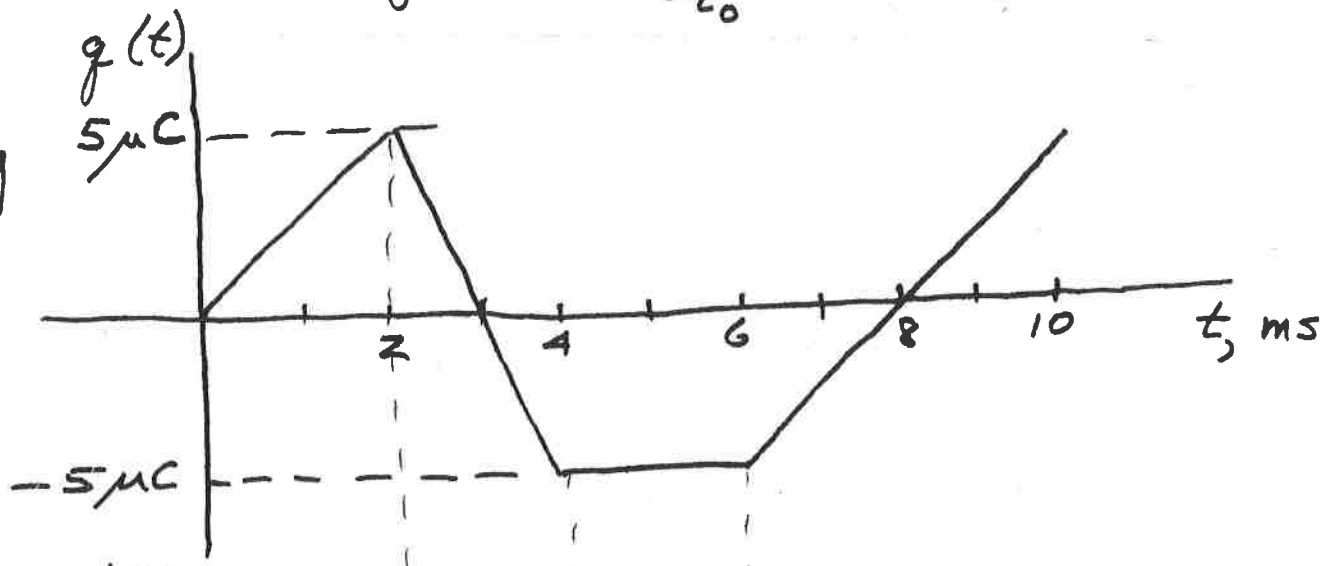
From $$i = \frac{dq}{dt}$$

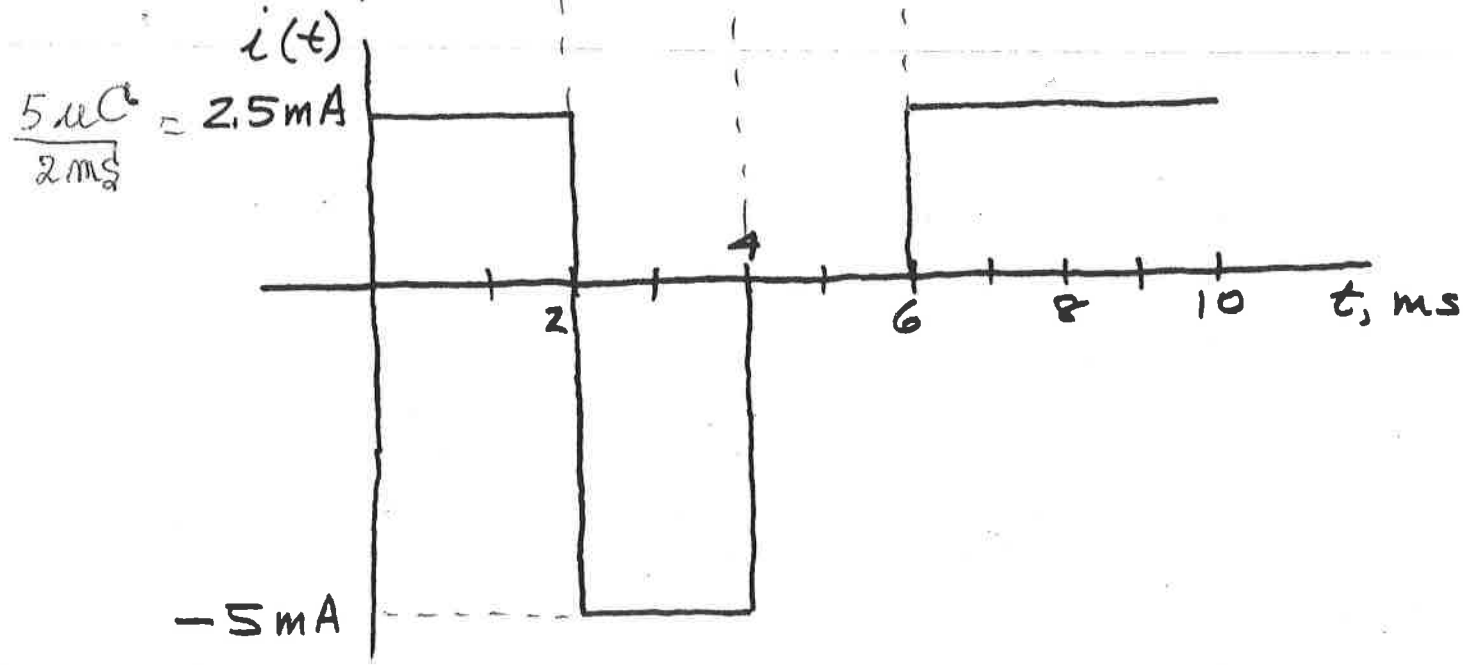
$$dq = i\,dt$$

$$\int_{t_0}^{t} dq = \int_{t_0}^{t} i\,dt$$

$$q(t) = \int_{t_0}^{t} i\,dt + q(t_0)$$

eg





Electric potential difference ("voltage")

Section 2.2 p. 14

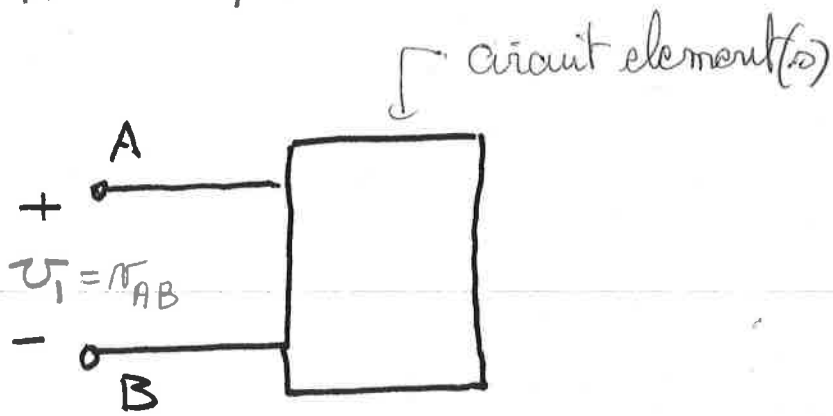
v_{AB} = "potential of A with respect to B"

= "potential drop from A to B"

= "potential rise from B to A"

= work per unit charge required to move a test charge from B to A (J/C or volts) (V)

1 volt = 1 J/C

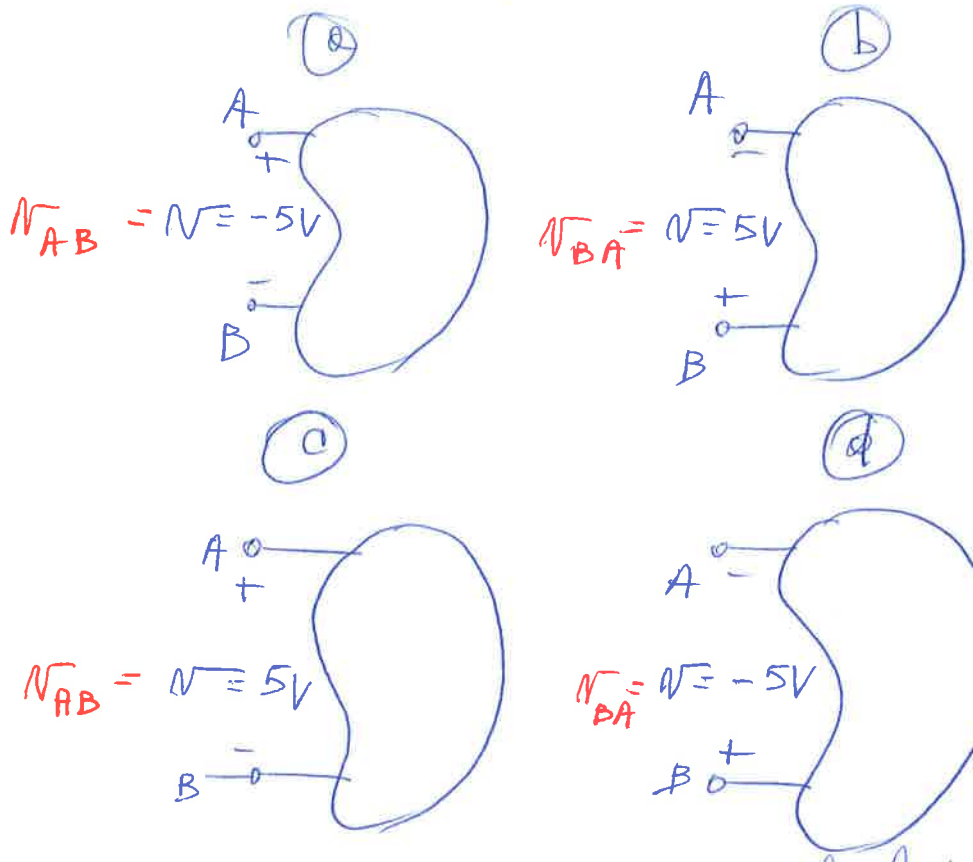


Alternate representations of a voltage:

$$v_1 = v_{AB} = -v_{BA}$$

+ − signs for v_1, and identification of points A and B for v_{AB} <u>must be shown</u>!

4 possibilities



a & b

Terminal B is 5 V positive with respect to terminal A

c & d

Terminal A is 5V positive with respect to terminal B

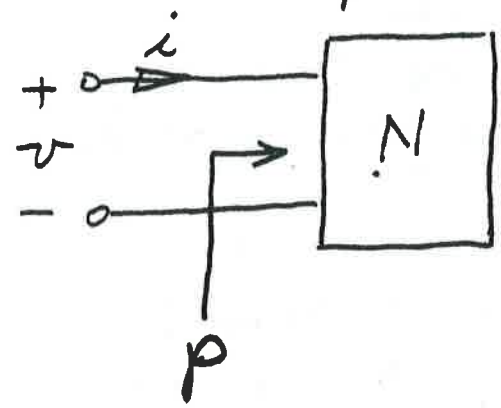
A convention is needed to distinguish between energy supplied <u>to</u> an element and energy supplied <u>by</u> an element

Passive Sign Convention

If a positive current is entering a terminal of the element and an external source must expand energy to establish this current, then terminal A is positive with respect to terminal B.

Alternatively, we may say that terminal B is negative with respect to terminal A.

1-6 Electric power



(Choice of symbols v and i corresponds to "passive sign convention")

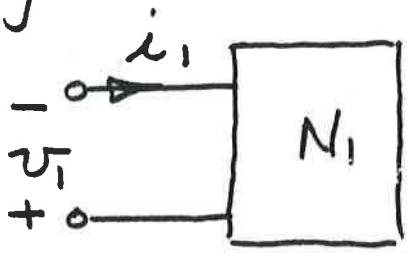
p = power delivered <u>to</u> the network N

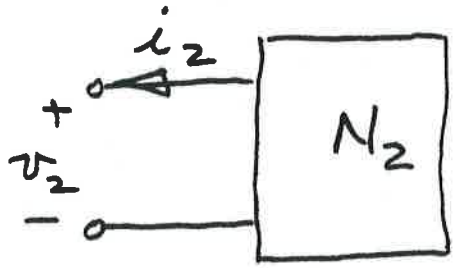
$$= \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt}$$

$$p = vi \quad (W)$$

The symbol p does not have intrinsic meaning; it must be unambiguously defined for every particular example.

eg





p_1 = power delivered to N_1

$$= -v_1 i_1$$

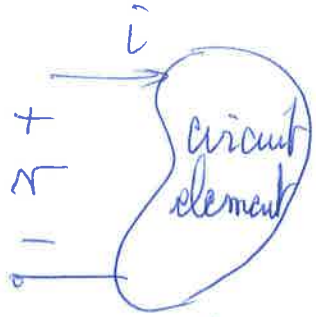
p_2 = power delivered <u>from</u> N_2

= − power absorbed by

$$= v_2 i_2$$

$\boxed{\text{Power } \left\{\begin{matrix}\text{delivered to}\\ \text{absorbed by}\end{matrix}\right\} \text{ N} = -\text{ power } \left\{\begin{matrix}\text{delivered by}\\ \text{absorbed by}\end{matrix}\right\} \text{ N}}$

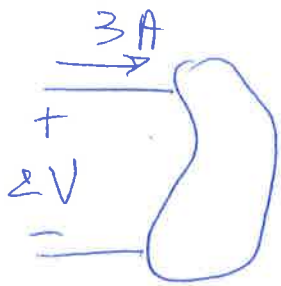
Figure 2.12



Current arrow is directed INTO the element at the plus marked terminal

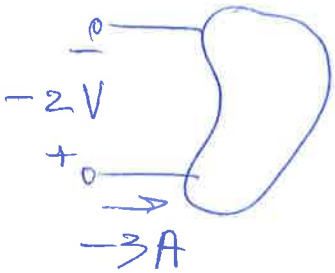
- The power absorbed by the element is $\boxed{p = vi}$. Alternatively, we can say that the element "generates" or "supplies" a power $\boxed{-vi}$
- Negative absorbed power = positive supplied power
- If one joule (1 J) of energy is expended in transferring one Coulomb of charge through the device in one second, than the rate of energy transfer is one Watt.

Example Fig 2.13 Computer power $\left\{\begin{matrix}\text{absorbed by} \\ \text{delivered to}\end{matrix}\right\}$ the circuit elements shown below.

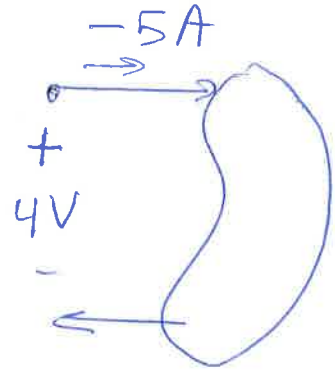


$P_{abs} = 2V \cdot 3A$
$= 6W$

are equivalent



$P_{abs} = (-3A)(-2V)$
$= 6W$



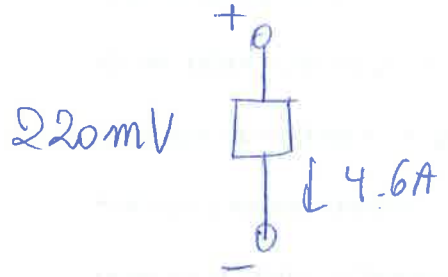
$P_{abs} = (4V)(-5A)$
$= -20W$

Negative absorbed power. The element is actually supplying +20W. The element is a source of energy

Do practice problems 2.6/2.7/2.8

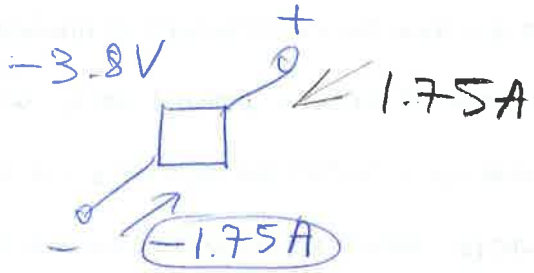
Practice problems P.17

2.6 Find power absorbed by circuit element below



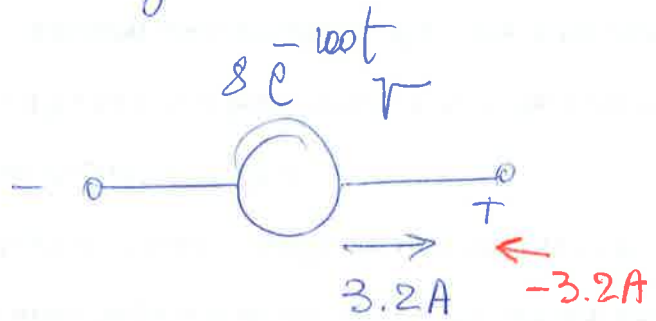
$P_{abs} = 220 \times 4.6 \times 10^{-3}\ W$
$= 1.012\ W$

2.7 Power delivered by circuit element below



power abs = $(-3.8V)(1.75A)$
power del = − power abs. by
$= 6.65\ W$

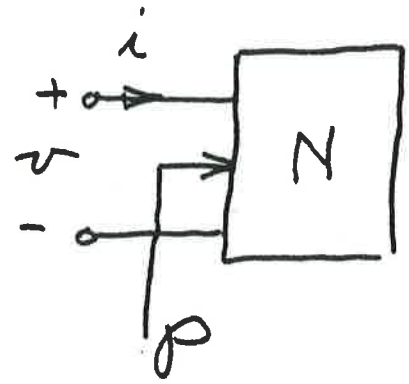
2.8 Find power being delivered to circuit element below at $t = 5ms$



$$\boxed{8e^{-100(5\times10^{-3})}\underbrace{(-3.2A)}_{\text{current entering + terminal}} = -15.53\ W}$$

[passive sign convention]

$$p = \frac{dw}{dt}$$



$$dw = p\,dt$$

$$\int_{t_0}^{t} dw = \int_{t_0}^{t} p\,dt'$$

$$w(t) = \int_{t_0}^{t} p\,dt' + w(t_0) \quad (J)$$

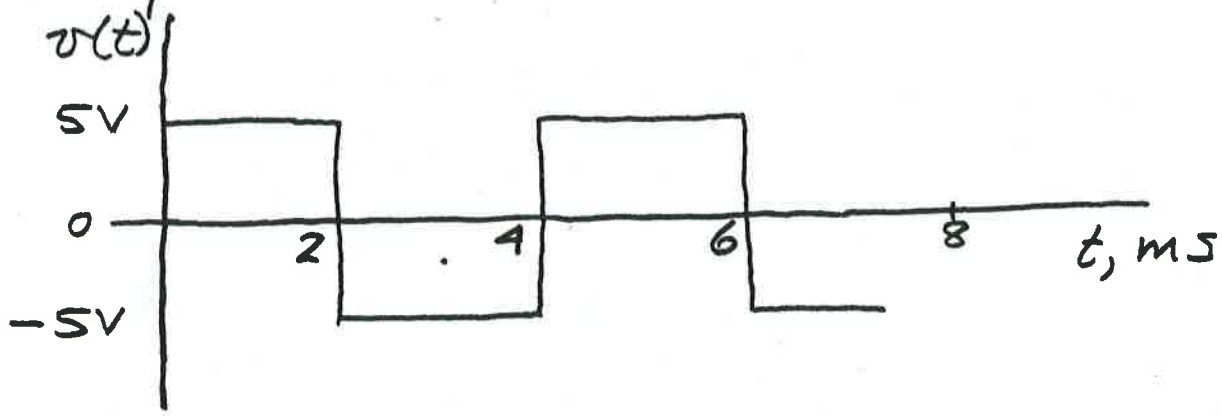
$$= \int_{t_0}^{t} v\,i\,dt' + w(t_0)$$

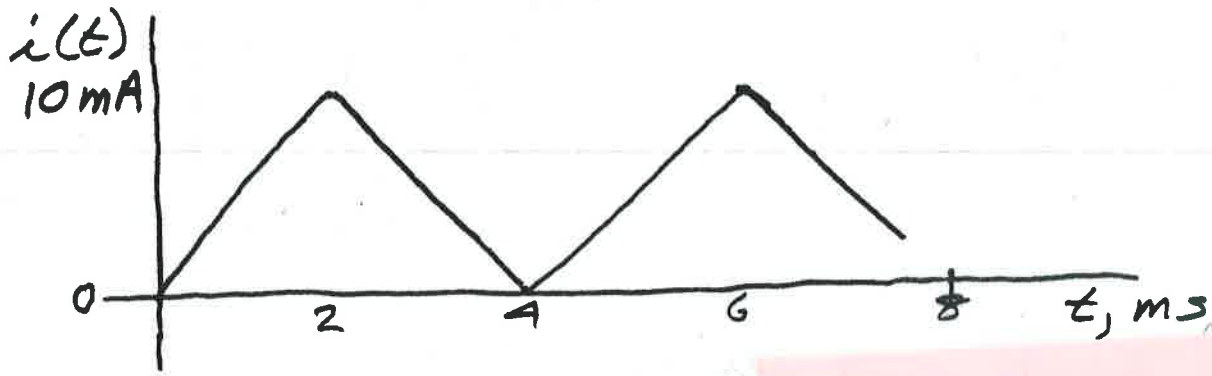
$\Delta w = w(t) - w(t_0) =$ energy delivered to N during the interval from $t = t_0$ until $t = t$

$$\boxed{\Delta w = w(t) - w(t_0) = \int_{t_0}^{t} v\,i\,dt'}$$

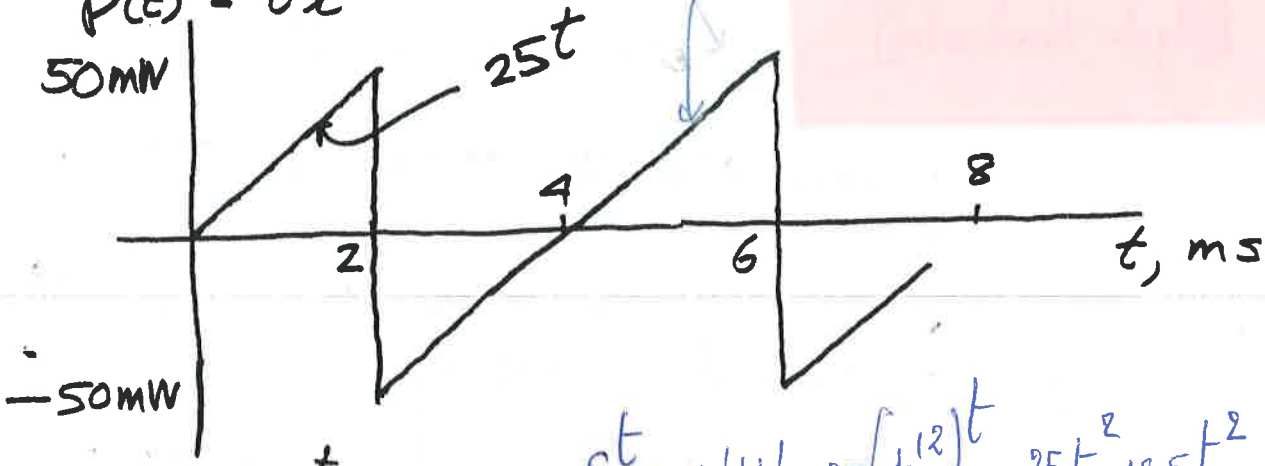
$=$ energy delivered to N during the interval from $t = t_0$ to $t = t$.

Example

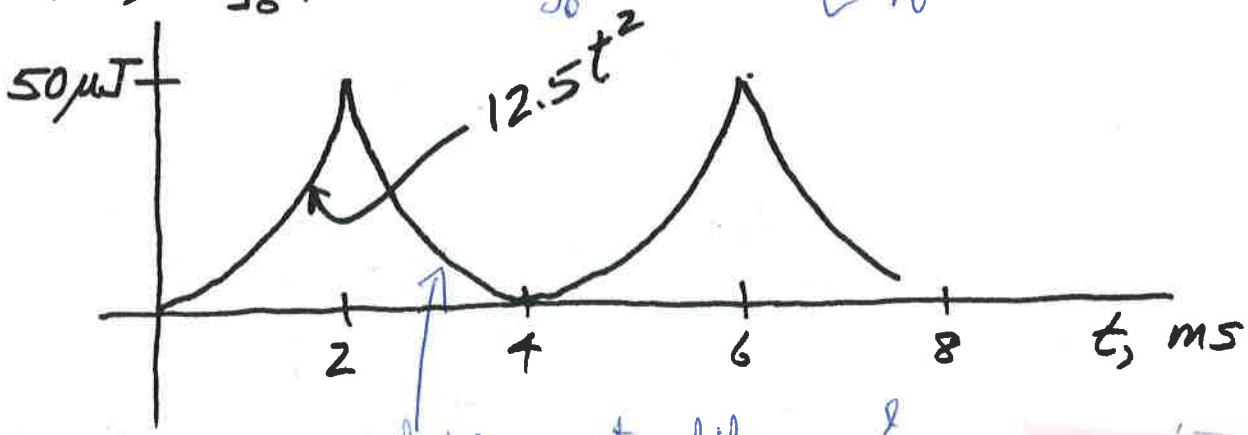




$p(t) = vi$

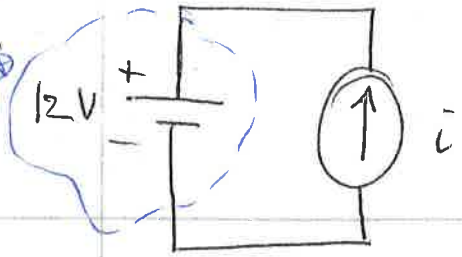


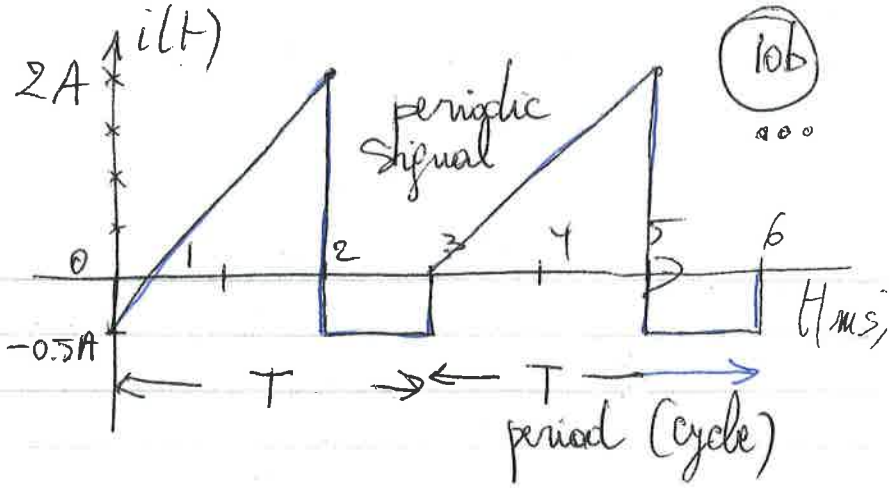
$$w(t) = \int_0^t p(t)\,dt = \int_0^t 25t'\,dt' = 25\left(\frac{t'^2}{2}\right)_0^t = \frac{25}{2}t^2 = 12.5t^2$$



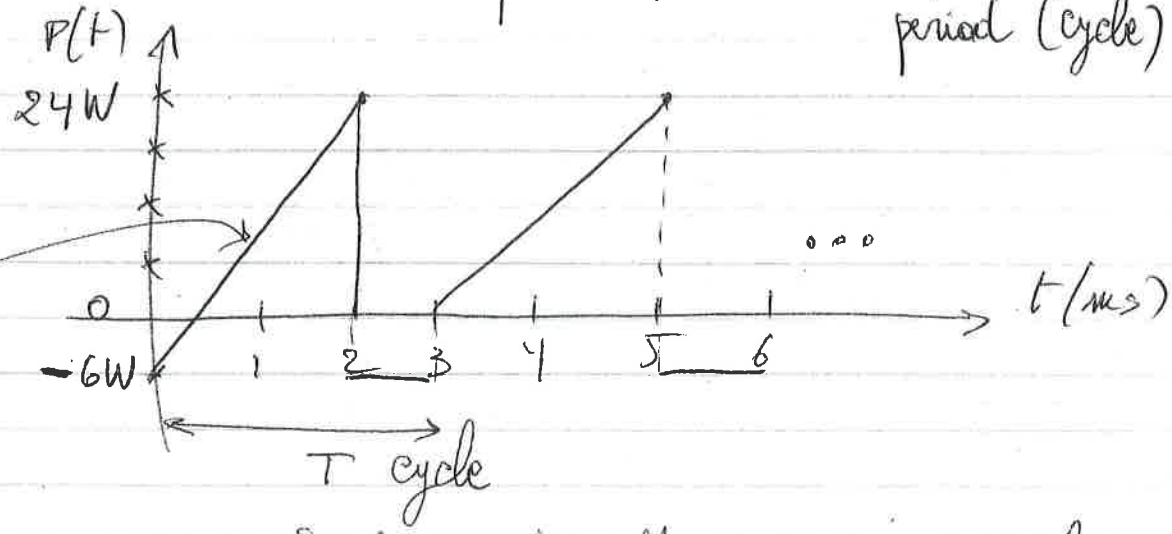
What is equation of this curve? in interval [2 ms – 6 ms]

circuit element





10b



Average power $\left\{\begin{array}{l}\text{delivered to}\\ \text{absorbed by}\end{array}\right.$ voltage source **per cycle**

Equation of line? $p(t)$

$i(t)$? between $[0,2]$ ms

$$i(t) - i(0) = \left[\frac{i(2\,ms) - i(0\,ms)}{2ms - 0ms}\right](t-0)$$

$$i(t) = -0.5 + \frac{2.5}{2\times10^{-3}}\,t = -0.5 + 1.25\times10^{3}\,t$$

in $[0,2]$ ms $\quad P(t) = v\,i(t) = 12(-0.5 + 1.25\times10^{3}\,t) = -6 + 15\times10^{3}\,t$

$$\frac{1}{T}\int_0^T p(t)\,dt = P_{av} = \frac{1}{3ms}\left[\int_0^{3ms} P(t)\,dt\right] = \frac{1}{3ms}\left[\int_0^{2ms}(-6+15\times10^{3}t)\,dt + \int_{2ms}^{3ms}(-6)\,dt\right]$$

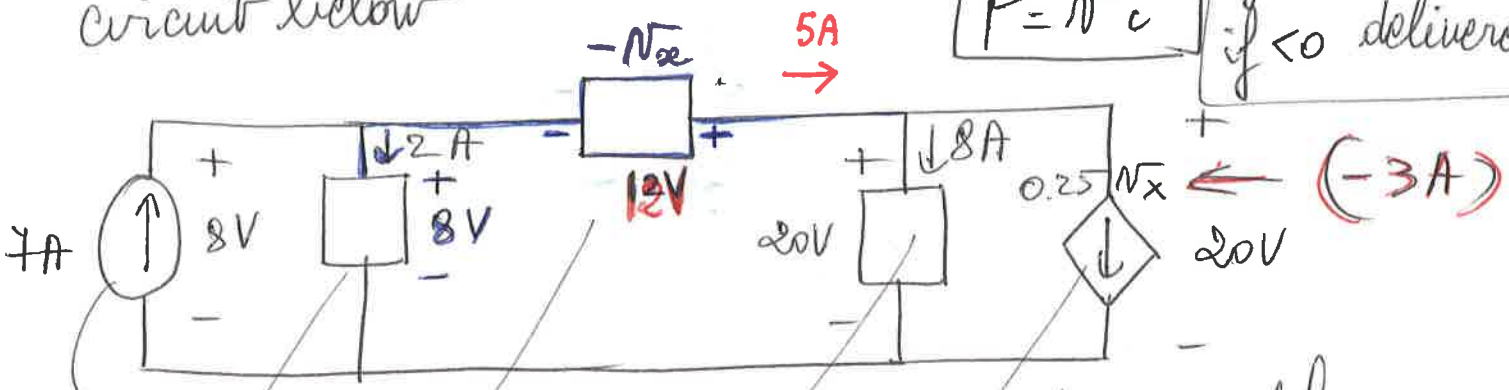
$$P_{av} = \frac{1}{3ms}\left[-6\times2ms + \frac{15\times10^{3}}{2}\left(t^2\right)_0^{2ms} - 6\times 1ms\right]$$

$$\boxed{P_{av} = \frac{1}{3ms}\left[-12ms + 30\,ms - 6ms\right] = 4\,W}$$

Practice Problem 2.9 (page 20)

Find the power absorbed by each element in the circuit below

$P = vi$ | if >0 absorbed by; if <0 delivered by



$8V(-7A) = -56W$, current source delivers 56W

$(8V)(2A) = 16W$ circuit element absorbs 16W

Element absorbs $-12V \cdot 5A = -60W$

absorbs 160W

absorbs $-60W$

Some elements deliver while other absorb power. Overall algebraic sum must be zero

Sum of power absorbed in circuit is

$$-56W + 16W - 60W + 160W - 60W = 0$$

as it should

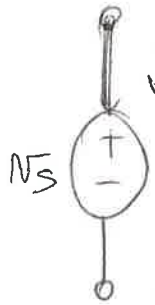
§ 2.3 p. 17 Voltage and Current Sources

By definition, a simple circuit element is the mathematical model of a two-terminal electrical device. It is completely characterized by its current-voltage characteristics. It can not be subdivided into other two-terminal devices.

Independent voltage sources

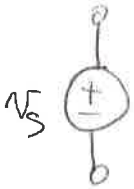
Symbol v_s

An independent voltage source is characterized by a terminal voltage which is completely independent of the current flowing through it. It is an ideal source.

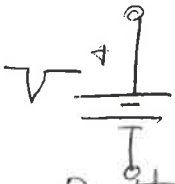


$\downarrow i \rightarrow$ passive sign convention is satisfied. The source absorbs the power $v_s i$.

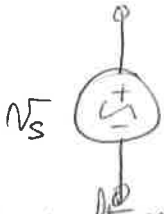
$\uparrow i \rightarrow$ power is delivered by the source.



DC voltage source symbol

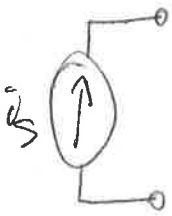


Battery symbol



AC voltage source symbol.

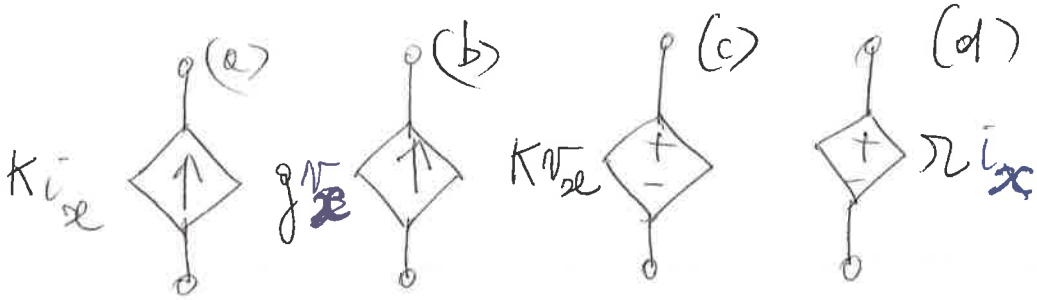
Independent Current Sources (p. 19)



The current through the element is completely independent of the voltage across it.

The voltage across is NOT zero necessarily. It can have any voltage across it!

Dependent Sources (p. 19)

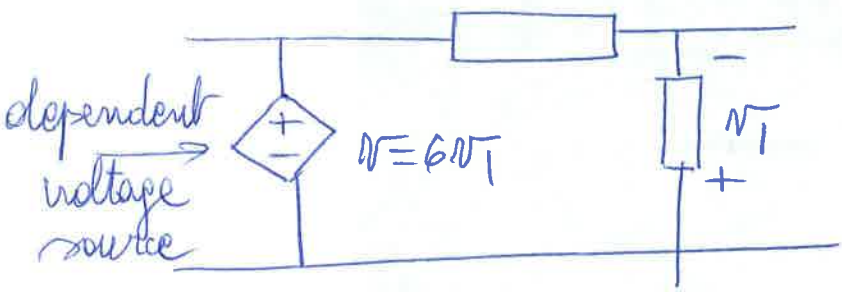


(a) Current-controlled current source.
(b) Voltage- " " " .
(c) Voltage- " voltage source.
(d) Current- " voltage source.

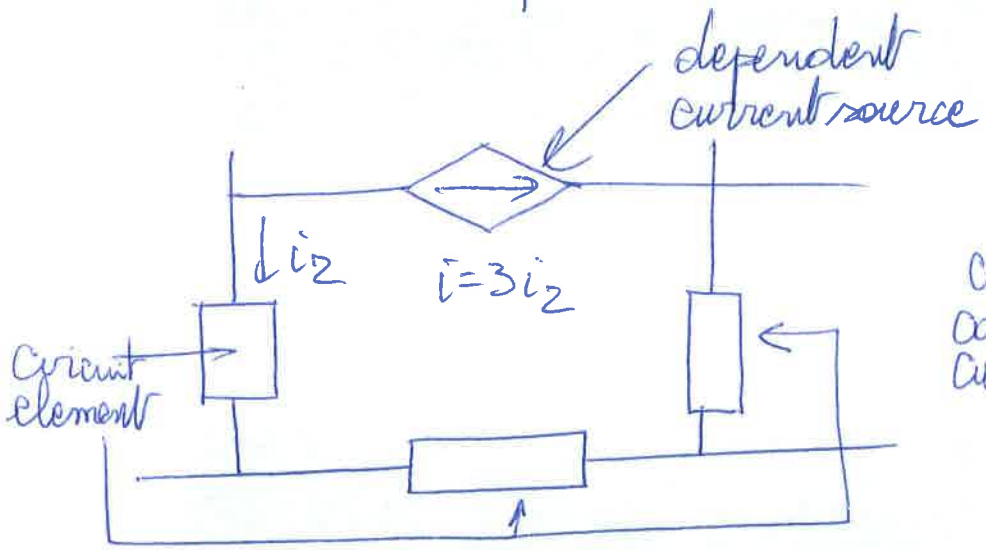
Such sources appear in the equivalent circuit models of many electronic devices (see transistors later).

In figures (a) and (c), K is dimensionless
In fig (b), g is a conductance.
In fig (d), r is a resistance.

Examples: dependent or controlled sources



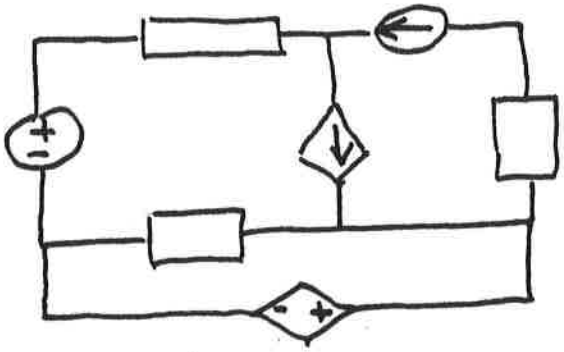
voltage controlled voltage source



current controlled current source

1-7 Networks and network components ("elements")

Electric network:
an interconnected set of (two-terminal) components



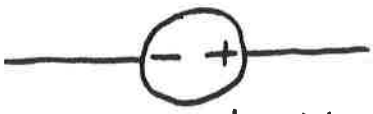
Components can be <u>passive</u> or <u>active</u>

<u>Passive</u> component: <u>average</u> power delivered to the component <u>must</u> be nonnegative

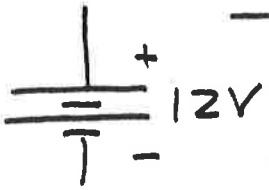
Resistor
Inductor

Capacitor
Transformer

<u>Active</u> component ("source"): <u>average</u> power delivered to the component <u>can</u> be negative



$v = 10 \sin 100t$ V



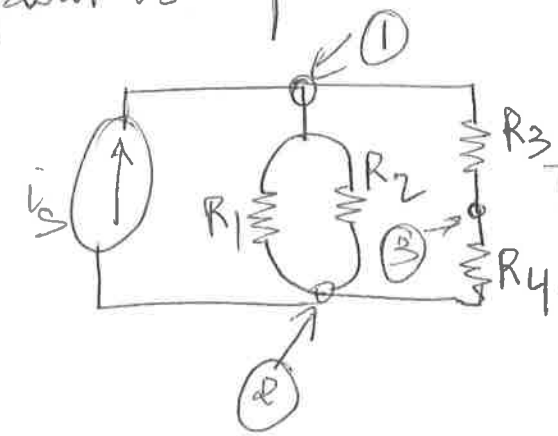
$i = 2.5$ mA

<u>Independent</u> sources

Do practice problems

- A point at which two or more elements have a common connection is called a node.
- Moving through a network between 2 nodes, if <u>NO</u> node is encountered more than once, then the set of nodes and elements that we have gone through is defined as a <u>PATH</u>. If the node at which we started is the same as the node at which we ended, the path is, by definition, a closed path or <u>LOOP</u>.
- <u>BRANCH</u> ≜ single path in a network, composed of one simple element and the node at each end of that element.
- A path is a particular collection of branches

<u>EX:</u>



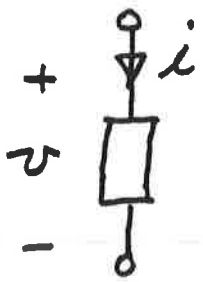
A network with 5 branches and 3 nodes

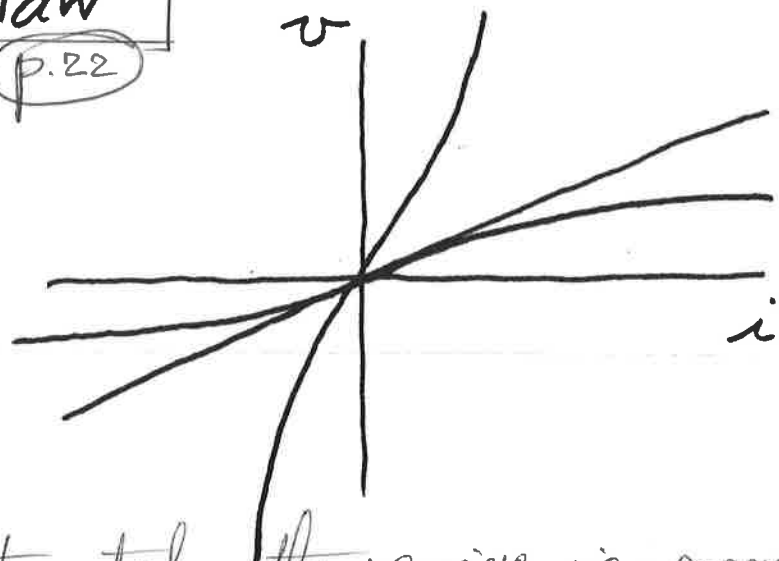
How many nodes and branches?

Chapter 2 Network Equations and Simple Networks

2-4 Ohm's "law"

p.22





v and i are selected to satisfy the passive sign convention.

For any two-terminal device, linear or nonlinear,

$$R = \frac{v}{i} \qquad V/A \text{ or ohms } (\Omega)$$

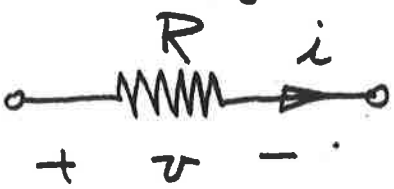
For a linear resistor, resistance R is <u>constant</u> — independent of v and i

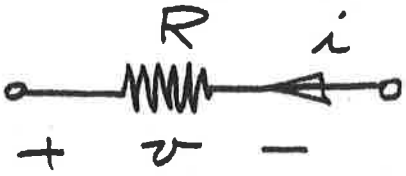
$$v = Ri$$

$$i = \frac{1}{R} v = Gv$$

Conductance $G = \frac{i}{v}$ $\quad A/V$ or mhos (℧) or siemens (S)

Network diagram representation:



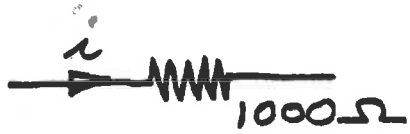


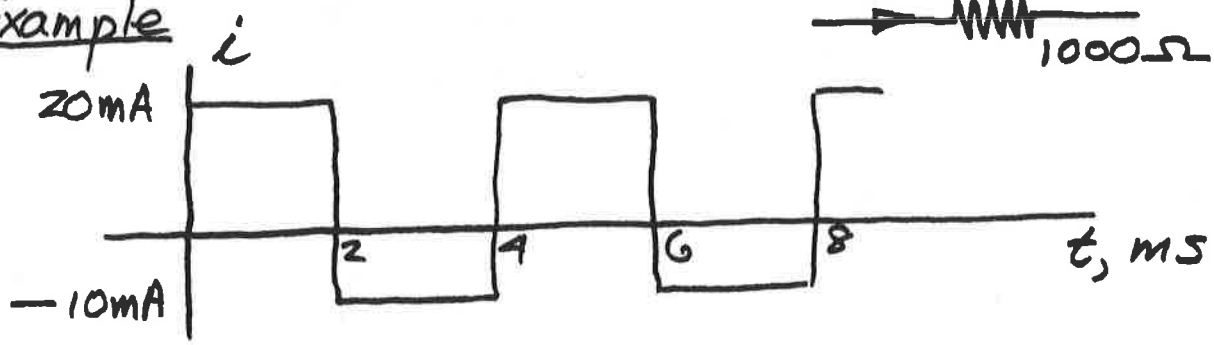
$v = Ri$ $\qquad\qquad$ $v = -Ri$

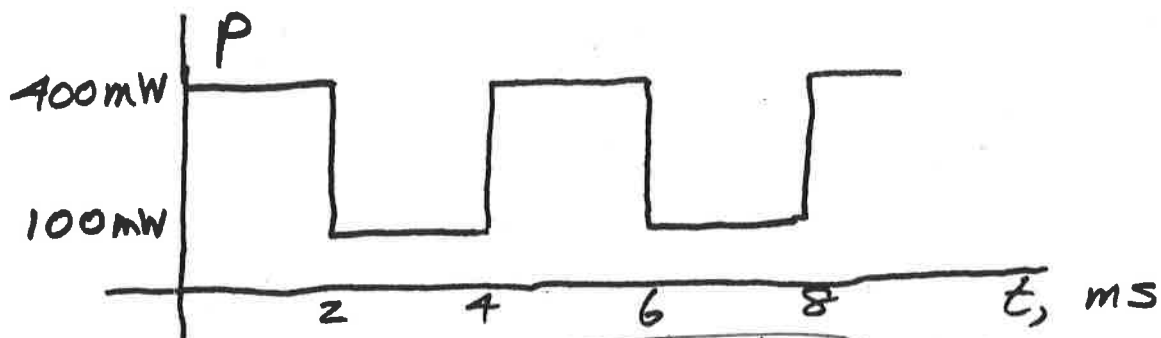
Power delivered to R:

$$p = vi \qquad\qquad p = -vi$$
$$= (Ri)i \qquad\qquad = -(-Ri)i$$
$$= Ri^2 \qquad\qquad = Ri^2$$
$$= \frac{v^2}{R} \leftarrow \text{agree} \rightarrow = \frac{v^2}{R}$$

<u>Example</u>





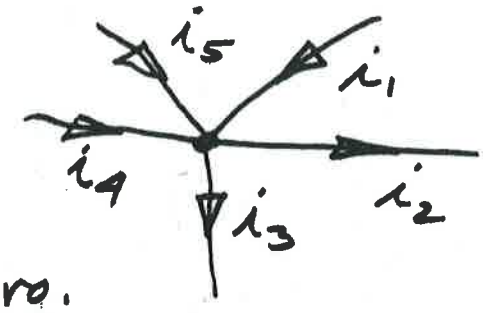


DO PRACTICE PROBLEMS 2.10/2.11/2.12 P.24

Chapter #3 Kirchhoff's laws (pp 39 → 78)

Kirchhoff's current law (KCL)

The (algebraic) sum of currents directed toward a junction (node) is zero.

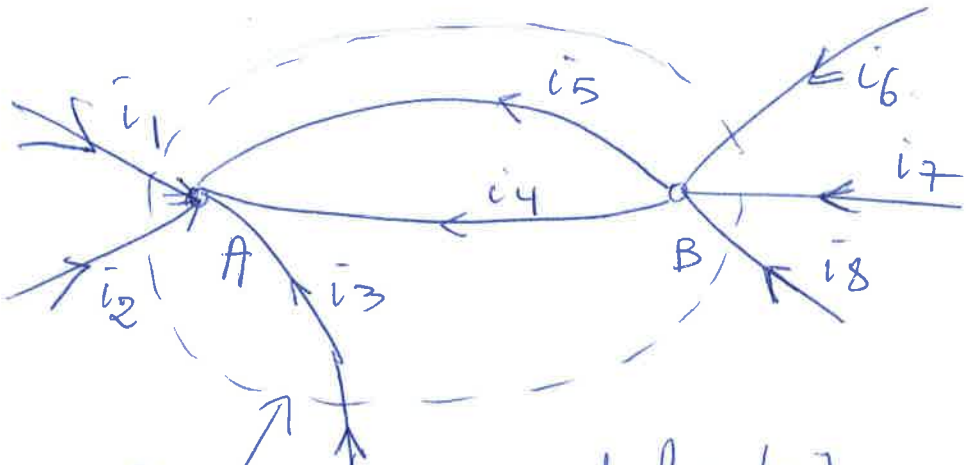


$$i_1 - i_2 - i_3 + i_4 + i_5 = 0$$

OR: $$-i_1 + i_2 + i_3 - i_4 - i_5 = 0$$

KCL is a network application of the principle of <u>conservation of charge</u>.

Generalization to supernodes



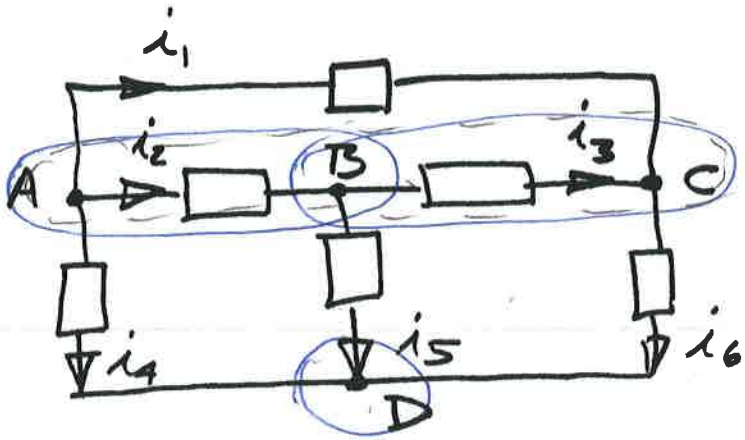
[supernode = any set of nodes]

at A: $i_1 + i_2 + i_3 + i_4 + i_5 = 0$

at B: $i_6 + i_7 + i_8 - i_4 - i_5 = 0$

For A and B together:

$$i_1 + i_2 + i_3 + i_6 + i_7 + i_8 = 0$$



Generalized KCL: the sum of currents directed toward <u>any set of nodes</u> is zero.

e.g.:

For AB: $-i_1 - i_3 - i_4 - i_5 = 0$

For D: $i_4 + i_5 + i_6 = 0$

For BC: $i_1 + i_2 - i_5 - i_6 = 0$

supernode

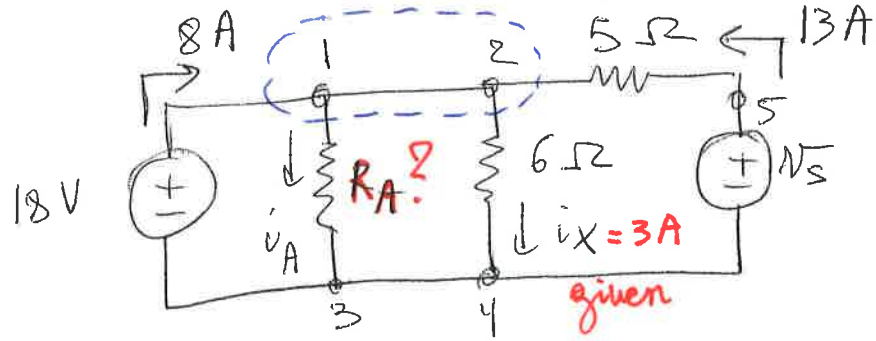
For ABC: $\cancel{i_1}\ \cancel{i_1} - i_4 - i_5 - i_6 = 0$

Practice problem 3.1 p. 38

Practice p. 38

Problem 3.1

Count the number of branches and nodes in the circuit below. If $i_X = 3A$ and the 18V source delivers 8A of current, what is the value of R_A?



number of branches?

① ≡ ②

③ ≡ ④

3 nodes / 5 branches

Unknown R_A

Given $i_X = 3A$

18 V source delivers 8 A.

Supernode

$i_A + i_X = 21$

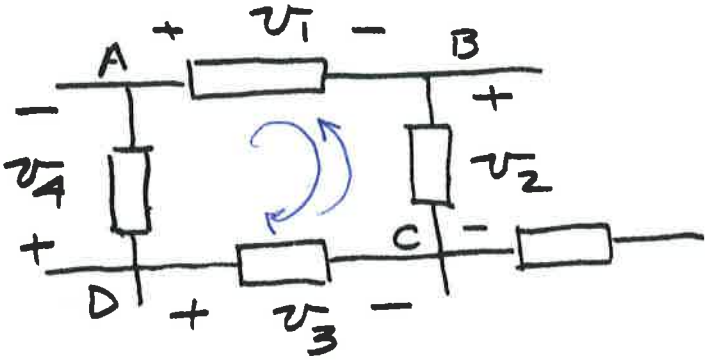
$i_A = 18A$

$R_A = 1\Omega \leftarrow 18 = R_A i_A$

$i_X = 3A$

Kirchhoff's voltage law (KVL)

The sum of all voltage drops (or rises) around any closed path is zero.



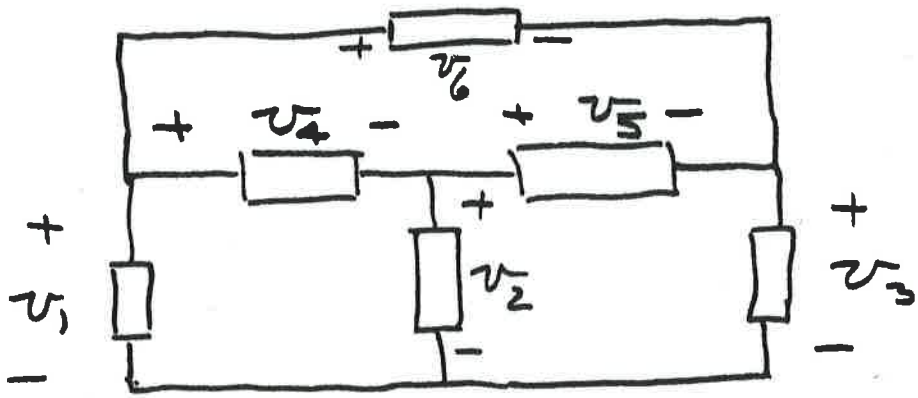
$$v_1 + v_2 - v_3 + v_4 = 0$$

OR $$-v_1 - v_2 + v_3 - v_4 = 0$$

OR $$v_{AB} + v_{BC} + v_{CD} + v_{DA} = 0$$

KVL is a network application of the principle of <u>conservation of energy</u>.

For a multi-loop network:



"Windows" or "Mesh" ≙ Loop that does not contain any other loop within it.

$$v_4 + v_2 - v_1 = 0$$

$$v_5 + v_3 - v_2 = 0$$

$$v_6 - v_5 - v_4 = 0$$

Other loops:

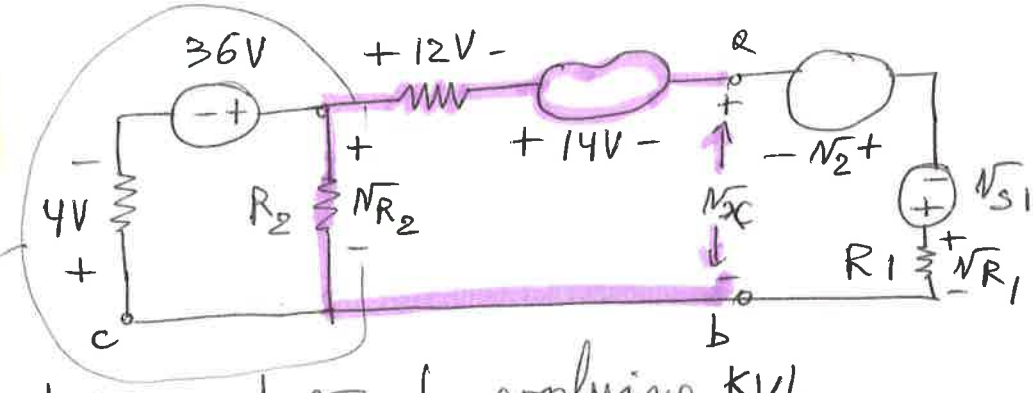
$$v_4 + v_5 + v_3 - v_1 = 0$$

$$v_6 + v_3 - v_1 = 0$$

by adding the 3 equations above

For a planar network with N nodes (junctions of 2 or more branches) and B branches (elements or components), the number of {windows / meshes} is

$$B - N + 1$$



Find v_{R_2} and v_x by applying KVL.

Going around this loop.

KVL $\quad 4 + v_{R_2} - 36 = 0$

$$\boxed{v_{R_2} = 32V}$$

Loop $c \to a \to b \to c$ going clockwise

$$4 - 36 + 12 + 14 + v_x = 0$$

$$\Rightarrow \boxed{v_x = 6V}$$

OR

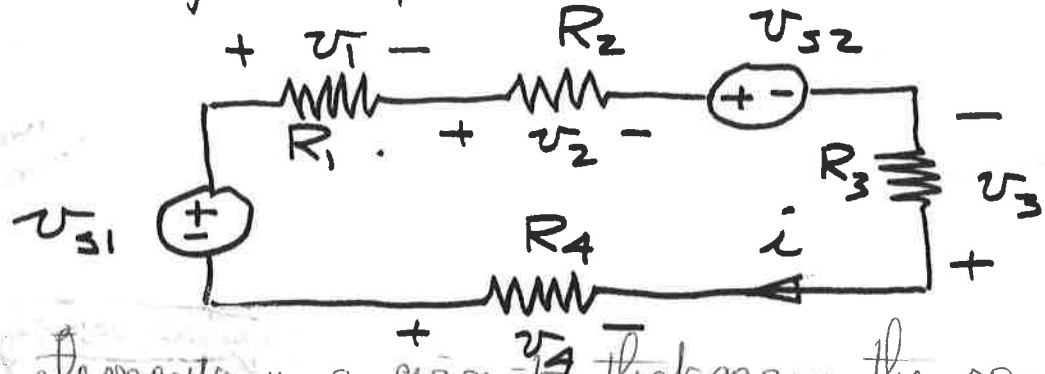
we know v_{R_2}.

Going around purple loop.

$$-32 + 12 + 14 + v_x = 0$$

$$\Rightarrow \boxed{v_x = 6V}$$

3-4 Single-loop networks



All of the elements in a circuit that carry the same current are said to be connected in series.

KCL : Same current i flows clockwise through each element.

KVL : $v_1 + v_2 + v_{s2} - v_3 - v_4 - v_{s1} = 0$

Component (Ohm's law) equations :

$$v_1 = R_1 i \qquad v_3 = -R_3 i$$
$$v_2 = R_2 i \qquad v_4 = -R_4 i$$

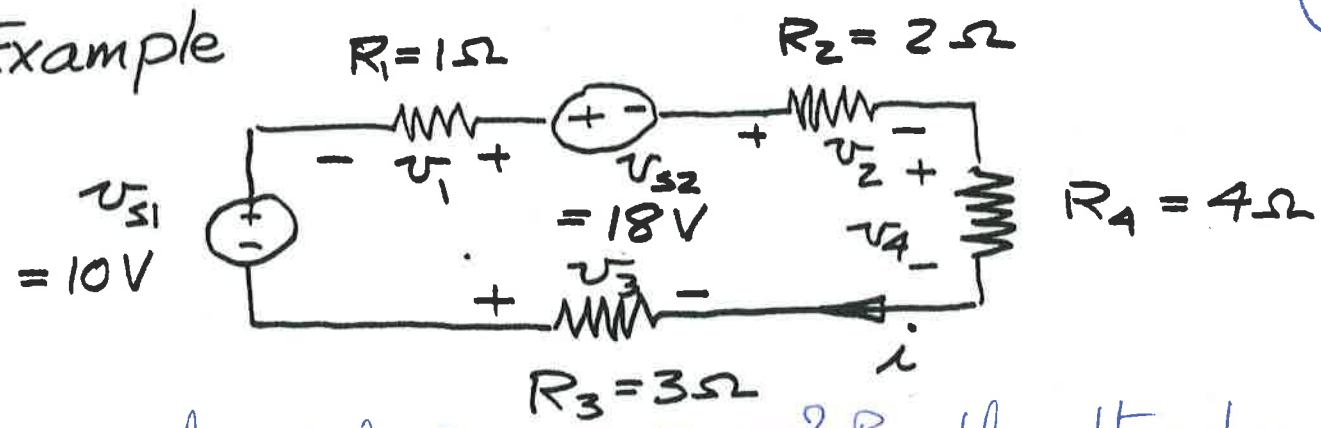
v_{s1} and v_{s2} are known and specified

Substitute :

$$R_1 i + R_2 i + v_{s2} + R_3 i + R_4 i - v_{s1} = 0$$

$$i = \frac{v_{s1} - v_{s2}}{R_1 + R_2 + R_3 + R_4}$$

Example



Questions: i through loop? v_1, v_2, v_3, v_4? Power delivered to each component?

$$v_{s2} + v_2 + v_4 - v_3 - v_{s1} - v_1 = 0$$

$$v_1 = -(1)i \qquad v_3 = -3i$$
$$v_2 = 2i \qquad v_4 = 4i$$

$$18 + 2i + 4i + 3i - 10 + i = 0$$

$$10i = 10 - 18 = -8$$

$$\longrightarrow i = -0.8\ A$$

Also: $v_1 = -i = 0.8\ V$ $\quad v_2 = 2i = -1.6\ V$
$v_3 = -3i = 2.4\ V$ $\quad v_4 = 4i = -3.2\ V$

Power delivered to each component:

$p_1 = -v_1 i = 0.64W$

$p_2 = v_2 i = 1.28W$

$p_3 = -v_3 i = 1.92W$

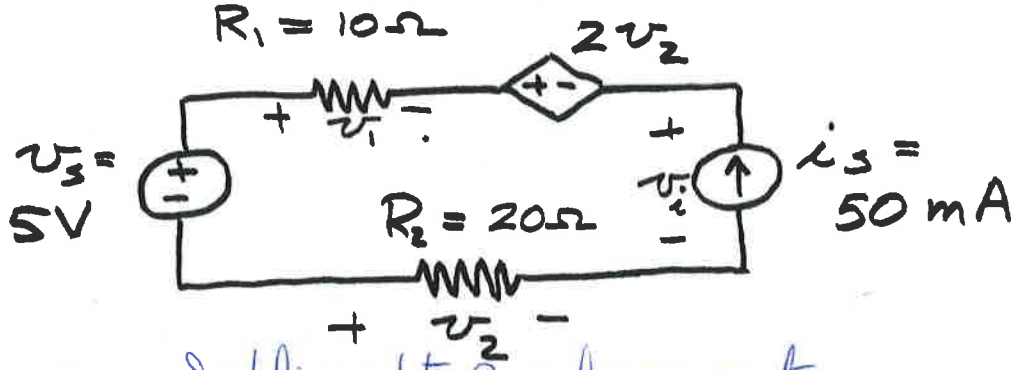
$p_4 = v_4 i = 2.56\ W$

$p_{s1} = -v_{s1} i = 8.0W$

$p_{s2} = v_{s2} i = -14.4\ W$

check: $p_1 + p_2 + p_3 + p_4 + p_{s1} + p_{s2} = 0$

EXAMPLE



Q: Power {delivered to / absorbed by} each component

Current is $i_s = 50$ mA throughout the loop

Ohm's law
$$\begin{cases} v_1 = -R_1 i_s = -0.5 \text{ V} \\ v_2 = R_2 i_s = 1.0 \text{ V} \end{cases}$$

$$v_1 + 2v_2 + v_i - v_2 - v_s = 0$$

$$v_i = v_2 + v_s - v_1 - 2v_2 = 4.5 \text{ V}$$

Power {delivered to / absorbed by} each component:

$$p_{R1} = -v_1 i_s = 25 \text{ mW}$$

$$p_{R2} = v_2 i_s = 50 \text{ mW}$$

$$p_{v_s} = v_s i_s = 250 \text{ mW}$$

$$p_{i_s} = -v_i i_s = -225 \text{ mW}$$

$$p_{2v_2} = -2v_2 i_s = -100 \text{ mW}$$

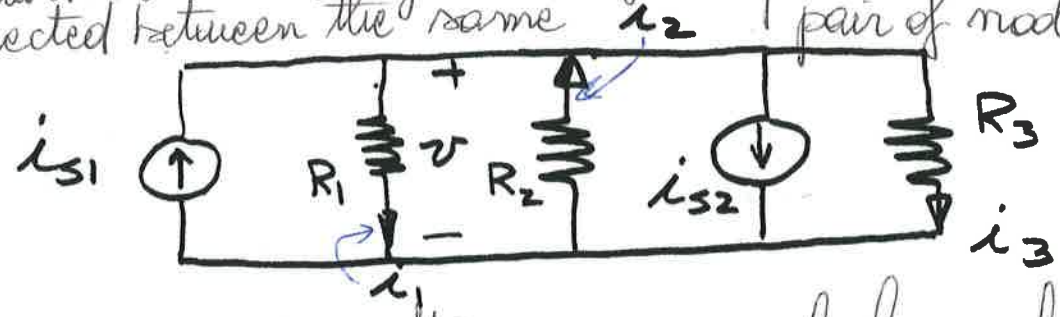
Sum = 0!

DO PRACTICE PROBLEMS 3.4 and 3.5

3-5 Single node-pair networks (p. 49)

network in which any number of simple elements are connected between the same pair of nodes.

i_{s1}, i_{s2} known



KVL : The voltage across each branch is the same as that across any other branch.

KCL : $-i_{s1} + i_1 - i_2 + i_{s2} + i_3 = 0$

Elements in a circuit having a common voltage across them are said to be connected in parallel

Component equations :

Ohm's law

Ohm

$$i_1 = \frac{v}{R_1}, \quad i_2 = \frac{-v}{R_2}, \quad i_3 = \frac{v}{R_3}$$

Substitute :

$$-i_{s1} + \frac{v}{R_1} + \frac{v}{R_2} + i_{s2} + \frac{v}{R_3} = 0$$

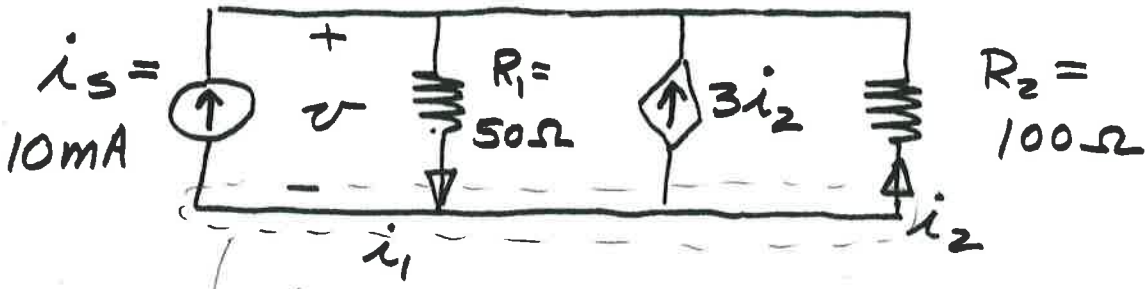
$$\longrightarrow \quad v = \frac{i_{s1} - i_{s2}}{1/R_1 + 1/R_2 + 1/R_3} = \frac{i_{s1} - i_{s2}}{G_1 + G_2 + G_3}$$

Find currents from component eqns, and

$$p_{R1} = v i_1$$

$$p_{is1} = -v i_{s1}, \text{ etc}$$

EXAMPLE [Power delivered to each component]



KCL: $-i_s + i_1 - 3i_2 - i_2 = 0$

Ohm's law $i_1 = \frac{v}{R_1} = \frac{v}{50}$, $i_2 = \frac{-v}{R_2} = \frac{-v}{100}$

Subst:

$$-10 + \frac{v}{50} + \frac{3v}{100} + \frac{v}{100} = 0$$

$$v\left(\frac{1}{50} + \frac{3}{100} + \frac{1}{100}\right) = 10$$

$$\longrightarrow v = \left(\frac{50}{3}\right)10 = 167 \text{ mV}$$

$$i_1 = \frac{v}{R_1} = 3.33 \text{ mA}$$

$$i_2 = \frac{-v}{R_2} = -1.67 \text{ mA}$$

Power delivered <u>to</u> each component:

$p_{R1} = v i_1 = 555 \mu W$ | $p_{is} = -v i_s = -1667 \mu W$

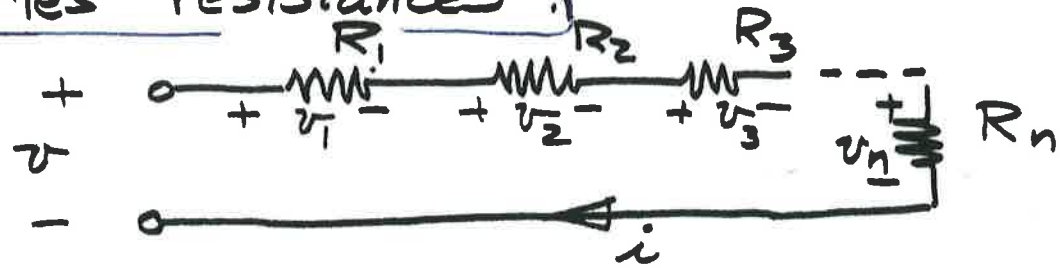
$p_{R2} = -v i_2 = 278 \mu W$ | $p_{3i_2} = -3 v i_2 = 833 \mu W$

$\boxed{\text{Sum Powers} = 0}$

DO practice problems 3.6 and 3.7

3-7 Resistance and source combinations (p. 55 book)

Series resistances :



KVL $v = v_1 + v_2 + --- + v_n$

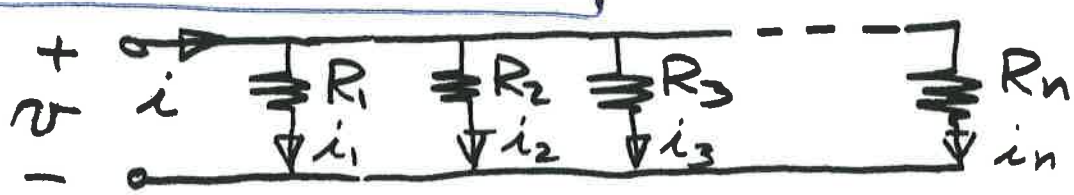
$$= R_1 i + R_2 i + --- + R_n i$$

$$= (R_1 + R_2 + --- + R_n) i$$

$$= R_{eq} i$$

$$\longrightarrow \quad R_{eq} = R_1 + R_2 + --- + R_n$$

Parallel resistances



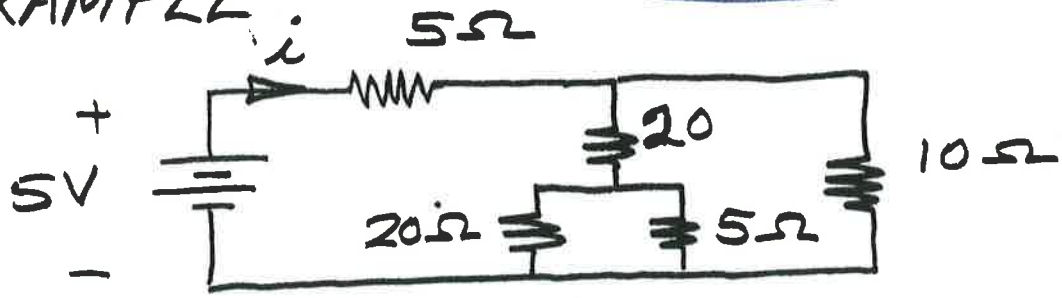
KCL $i = i_1 + i_2 + --- + i_n = \frac{v}{R_1} + \frac{v}{R_2} + --- + \frac{v}{R_n}$

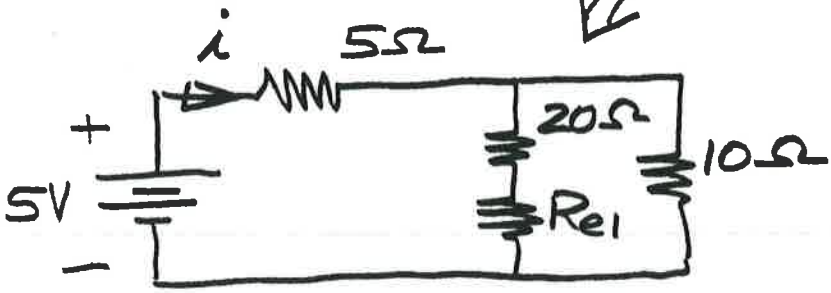
$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + --- + \frac{1}{R_n}\right) v = \frac{1}{R_{eq}} v$$

$$\longrightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + --- + \frac{1}{R_n} \quad (\text{OR}) \; G_{eq} = G_1 + G_2 + --- + G_n$$

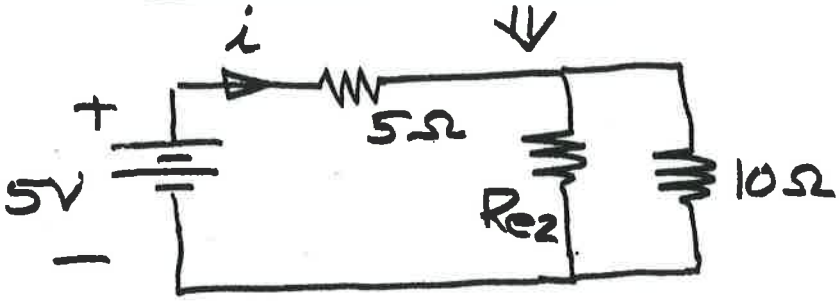
EXAMPLE

Find current i

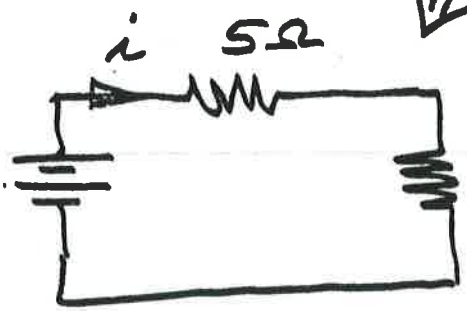




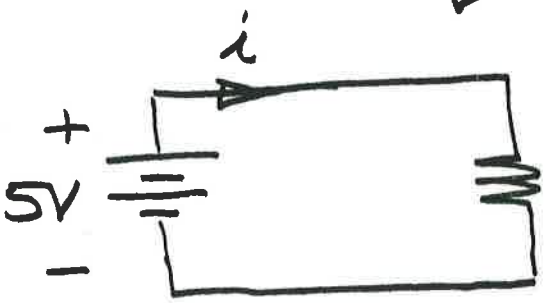
$$R_{e1} = \frac{1}{\frac{1}{20} + \frac{1}{5}} = \frac{(5)(20)}{5+20} = 4\,\Omega$$



$$R_{e2} = 20 + R_{e1} = 24\,\Omega$$



$$R_{e3} = \frac{(R_{e2})(10)}{R_{e2}+10} = 7.06\,\Omega$$



$$R_{e4} = 5 + R_{e3} = 12.06\,\Omega$$

$$i = \frac{5}{12.06} = 0.415\ A$$