ANSWER KEY for First Mid-Term
Computer Science 545
May 6, 1998

No books or notes. The time duration is **50** minutes. The exam is worth **100** points. Use back or extra sheets if needed.

Name:__________________________________ **SSN:**______________________

1. Give the transition graph for a dfa that accepts the following languages.

   (a) $L = \{w \in \{a,b\}^* : |w| = 0 \ mod\ 5\}$
   Create a directed cycle with 5 nodes. Each arc labeled with both a,b. Pick one node as both start and final state.

   (b) $L = \{w \in \{a,b\}^* : (n_a(w) - n_b(w)) = 0 \ mod\ 5\}$
   Create a directed cycle with 5 nodes. Each arc labeled with a. Add another cycle with reverse orientation and label each arc with b. Pick one node as both start and final state.

2. Given two dfas M_1 and M_2, is it possible to test if the languages they accept are the same, i.e., $L(M_1) = L(M_2)$? How?

   Yes, first construct a dfa for the language $L = L(M_1) \cap \bar{L}(M_2) \bigcup L(M_2) \cap \bar{L}(M_1)$. The method of construction for union and intersection of languages was described in class. Now $L(M_1) = L(M_2)$ iff $L = \phi$. To test whether $L = \phi$, we simply test whether there is a path from the start state to a final state in the transition graph of the dfa.

3. Under what condition are two states p, q in a dfa $M = (Q, \Sigma, q_0, \delta, F)$ considered to be *indistinguishable*.

   Two states p and q are indistinguishable if for any string w, the w-walk from p ends in a final state implies the w-walk from q ends in a final state, and the w-walk from p ends in a nonfinal state implies the w-walk from q ends in a nonfinal state.

4. Give a regular expression for $L = \{w \in \{a,b\}^* : w$ does not contain aa as a substring $\}$

   $(b + ab)^*(\lambda + a)$

5. Using alternating quantifiers $\forall, \exists$ give the statement of the Pumping Lemma for regular languages.

   $\forall$ regular languages L
   $\exists$ positive integer m such that
   $\forall$ strings w with $|w| \geq m$
   $\exists$ a decomposition $w = xyz$ where $|xy| \leq m, |y| \geq 1$ such that
   $\forall$ integers $i \geq 0$,
   $\exists$ $w_i = xy^iz \in L$

6. Apply this to show that $L = \{0^n w : w \in \{0,1\}^*, |w| < n\}$ is not regular.

   Assume L is regular. Take m as given in the statement of the Pumping Lemma. Choose $w = 0^m 1^{m-1}$. Any decomposition $w = xyz$ where $|xy| \leq m, |y| \geq 1$ results in y being a nonempty string of 0s. Now, $w_0 = xz = 0^{m-k}1^{m-1} \notin L$ since $m - k \leq m - 1$. This is a contradiction, hence L cannot be regular.

7. Give a UNIX egrep command that finds all words in /usr/dict/words that begin with letter c and contains either substring ab or ba,

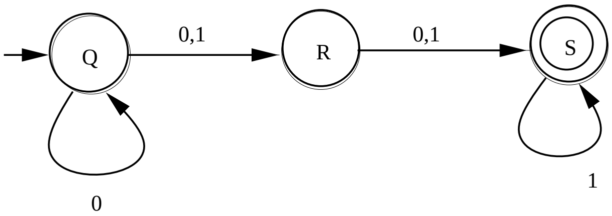
```
egrep '^c.*(ab|ba).*$' /usr/dict/words
```

8. Construct a dfa that accepts the language generated by the grammar

$$S \to abA, A \to baB, B \to aA|bb.$$

```
                ________<a_______
                |               |
S ----> o ----> A ----> o ----> B ----> o ----> F
   a       b        b       a       b       b
```

9. Construct the transition graph for a dfa that is equivalent to the following nfa.



Here is a sketch of an equivalent dfa.

```
      0          0       0
Q------>----QR--->-----QRS
|           |1        |
|1          |         |1
v   <----- RS--------<
|  /0       |
| /         v1
R----->-----S
       0,1   \1
```